Pushing to the Top with K-induction

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Agenda

IC3 is one of the most powerful algorithms for model checking safety properties

Very active area of research:

- A. Bradley: *SAT-Based Model Checking Without Unrolling.* VMCAI 2011
  (IC3 stands for "Incremental Construction of Inductive Clauses for Indubitable Correctness")

- N. Eén, A. Mishchenko, R. Brayton: *Efficient implementation of property directed reachability.* FMCAD 2011
  (PDR stands for "Property Directed Reachability")

  …

- In this talk, I present a new IC3-based algorithm, called QUIP
  (QUIP stands for "a QUest for an Inductive Proof")

- **and** show how QUIP can be extended with k-induction

A brief preview of Quip

Quip extends IC3 by allowing for

- *A wider range of conjectures (proof obligations)*
  - Designed to push already existing lemmas more aggressively
  - Allows to push a given lemma by learning additional *supporting* lemmas
    (and hopefully to compute an inductive invariant faster)
- *Forward reachable states*
  - Explain why a lemma cannot be pushed
  - Allows to keep the number of proof obligations under control

These are integrated into a single algorithmic procedure

The experimental results look good

Problem: Symbolic Safety and Reachability

A transition system P = (V, Init, Tr, Bad)

P is UNSAFE if and only if there exists a number N s.t.

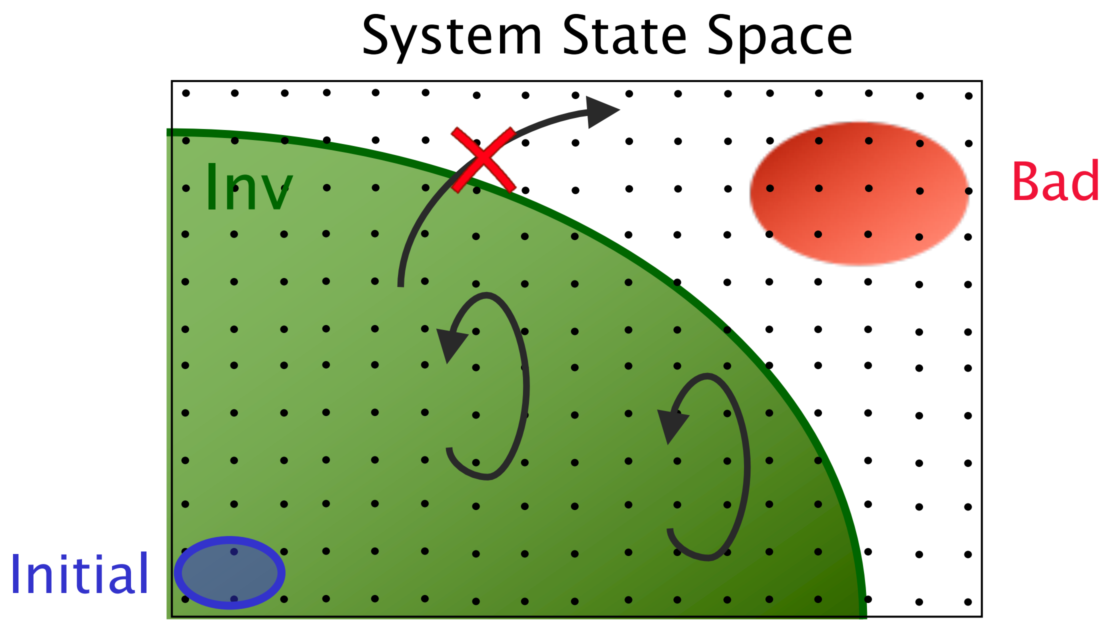
$$Init(X_0) \wedge \left(\bigwedge_{i=0}^{N-1} Tr(X_i, X_{i+1}) \right) \wedge Bad(X_N) \not\Rightarrow \perp$$

P is SAFE if and only if there exists a safe inductive invariant Inv s.t.

$$\left.\begin{array}{r} Init \Rightarrow Inv \\ Inv(X) \wedge Tr(X, X') \Rightarrow Inv(X') \end{array}\right\} \boxed{\text{Inductive}}$$

$$Inv \Rightarrow \neg Bad \qquad \boxed{\text{Safe}}$$

Inductive Invariants



System S is safe iff there exists an **inductive invariant** Inv:

- Initiation: Initial $\subseteq$ Inv
- Safety: Inv $\cap$ Bad = $\emptyset$
- Consecution: TR(Inv) $\subseteq$ Inv

i.e., if s $\in$ Inv and s$\rightsquigarrow$t then t $\in$ Inv

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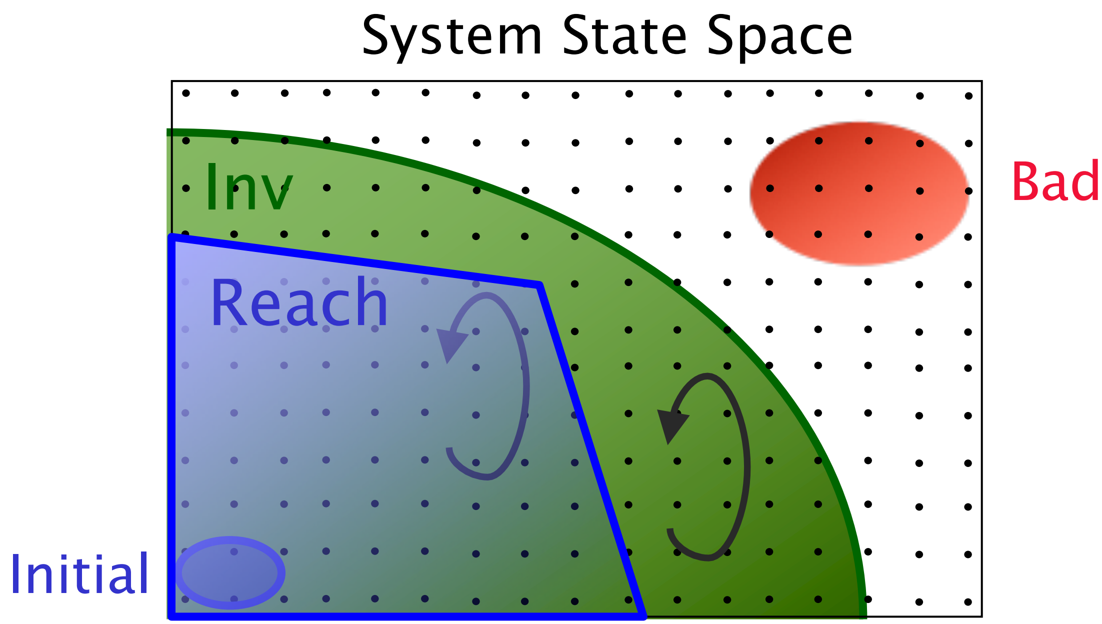
Inductive Invariants



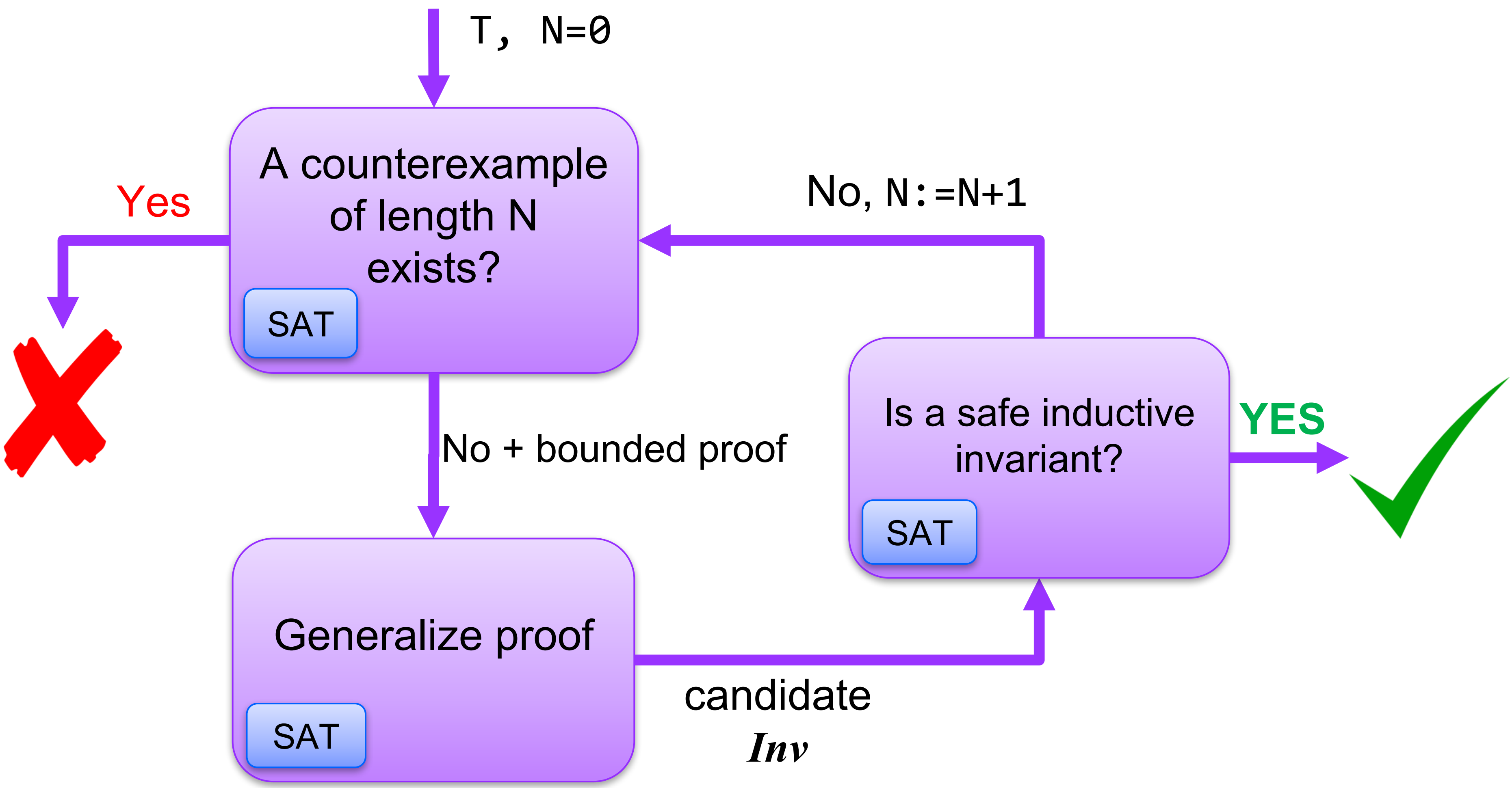
System S is safe iff there exists an **inductive invariant** Inv:

- Initiation: Initial ⊆ Inv
- Safety: Inv ∩ Bad = ∅
- Consecution: TR(Inv) ⊆ Inv

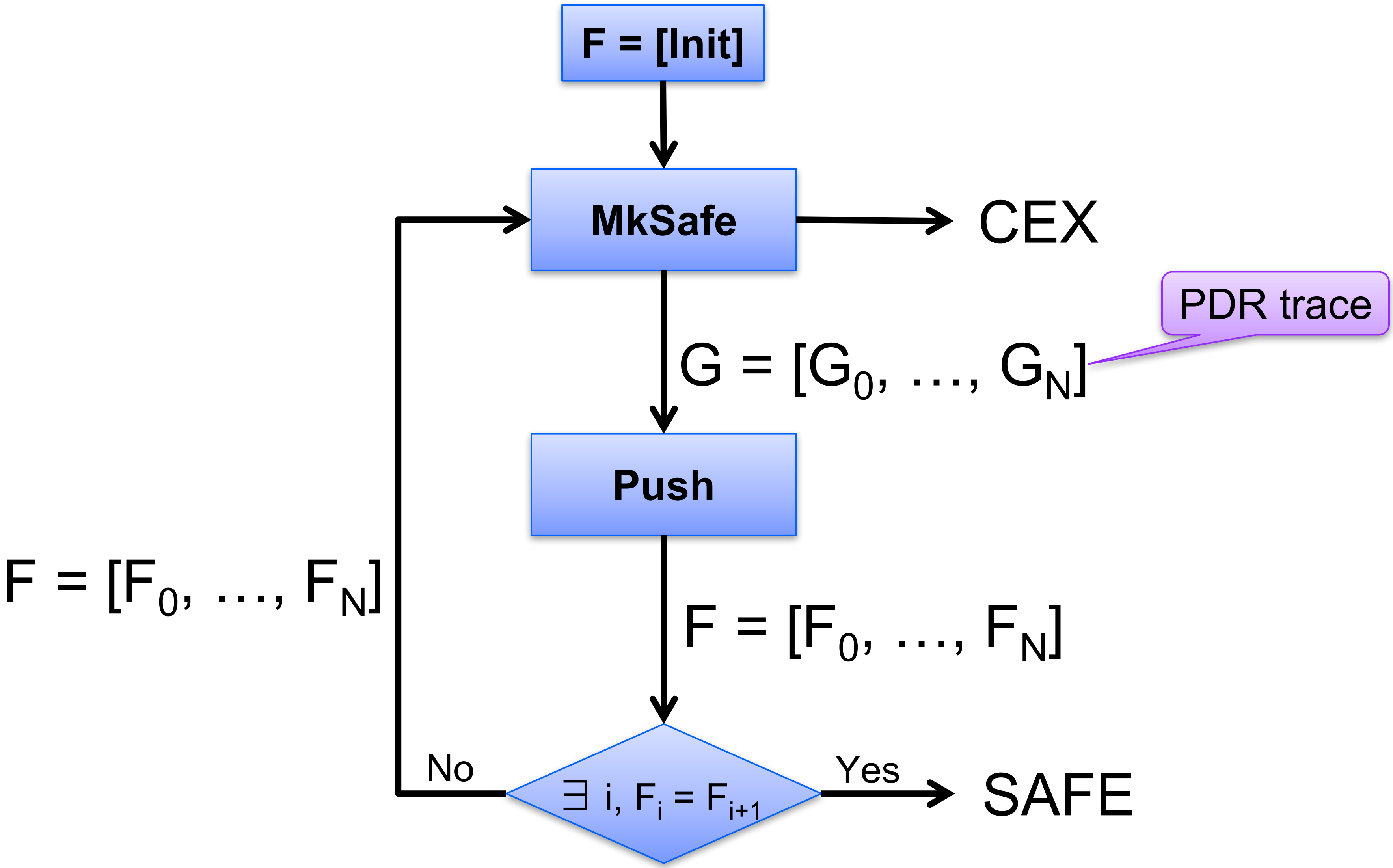
i.e., if s ∈ Inv and s⇝t then t ∈ Inv

System S is **safe** if Reach ∩ Bad = ∅

Generalizing from Bounded Proofs



IC3/PDR



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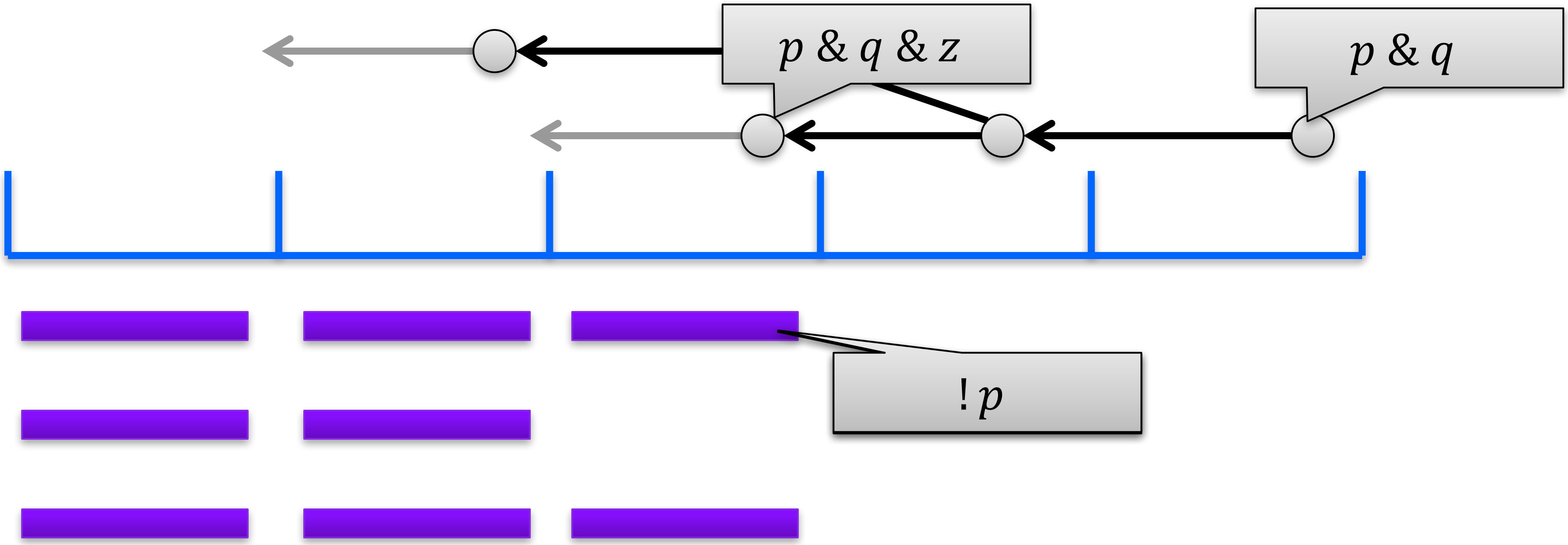
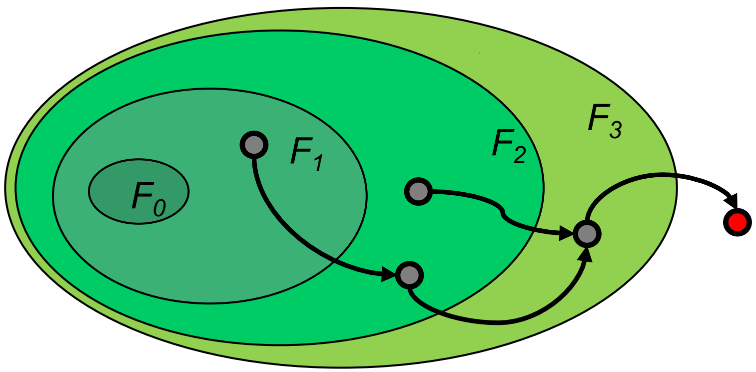
IC3/PDR In Pictures: MkSafe

MkSafe





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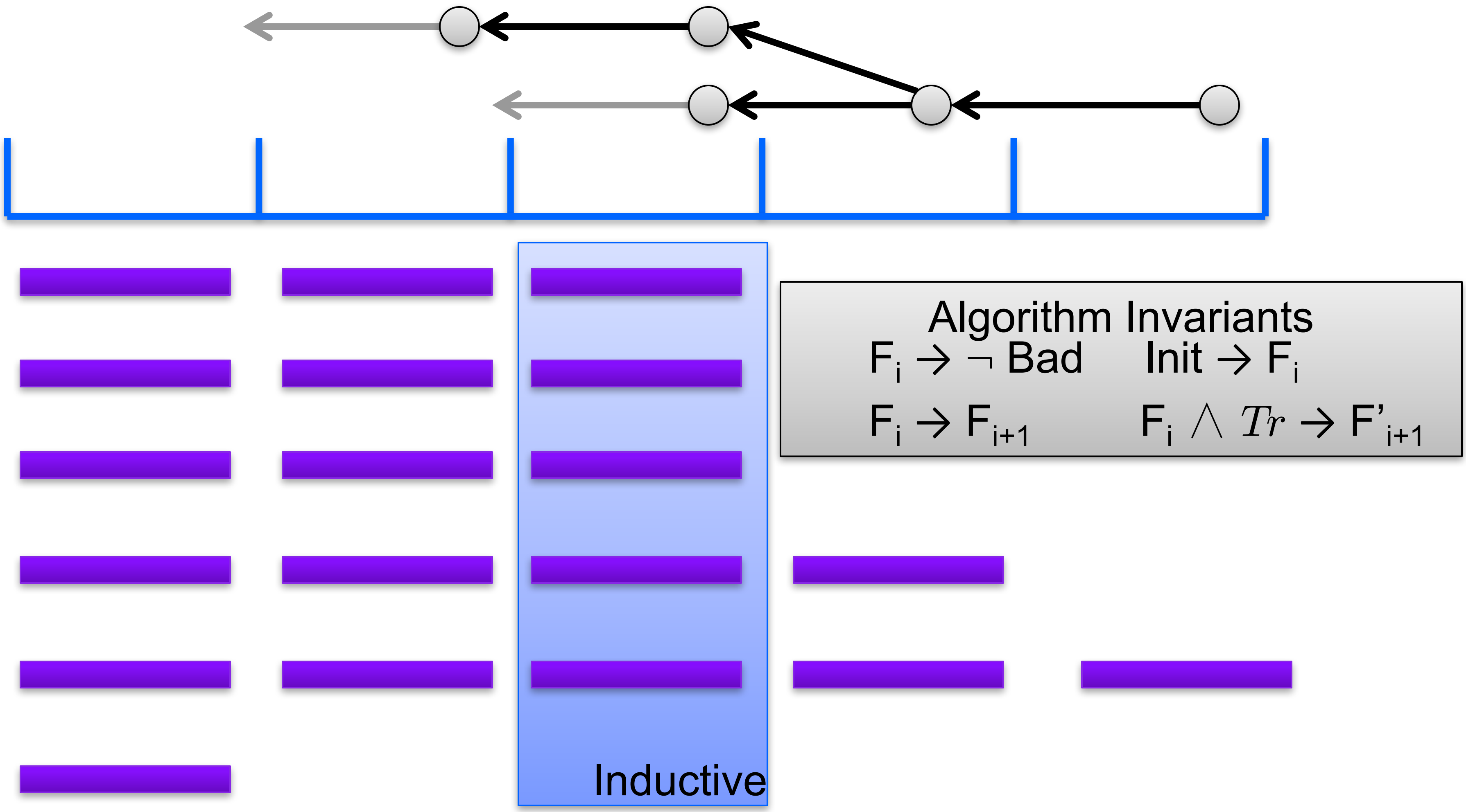
IC3/PDR in Pictures: Push

Push



Algorithm Invariants

$F_i \rightarrow \neg$ Bad Init $\rightarrow F_i$

$F_i \rightarrow F_{i+1}$ $F_i \wedge Tr \rightarrow F'_{i+1}$

A quick review of IC3/PDR

Input:

- A safety verification problem (Init, Tr, Bad)

Output:

- A counterexample (if the problem is UNSAFE),
- A safe inductive invariant (if the problem is SAFE)
- Resource Limit

Main Data-structures:

- A current working level N
- An *inductive trace*
- A set of *proof obligations*

Inductive Trace

Let $F_0, F_1, F_2, \ldots, F_\infty$ be conjunctions of lemmas (in practice, clauses).
We say that $F_0, F_1, F_2, \ldots, F_\infty$ is an *inductive trace* if:

(1) $F_0 = INIT$

(2) $F_0 \Rightarrow F_1 \Rightarrow F_2 \Rightarrow \ldots \Rightarrow F_\infty$ *(monotone)*

(3) $F_1 \supseteq F_2 \supseteq \ldots \supseteq F_\infty$ as sets of lemmas *(s. monotone)*

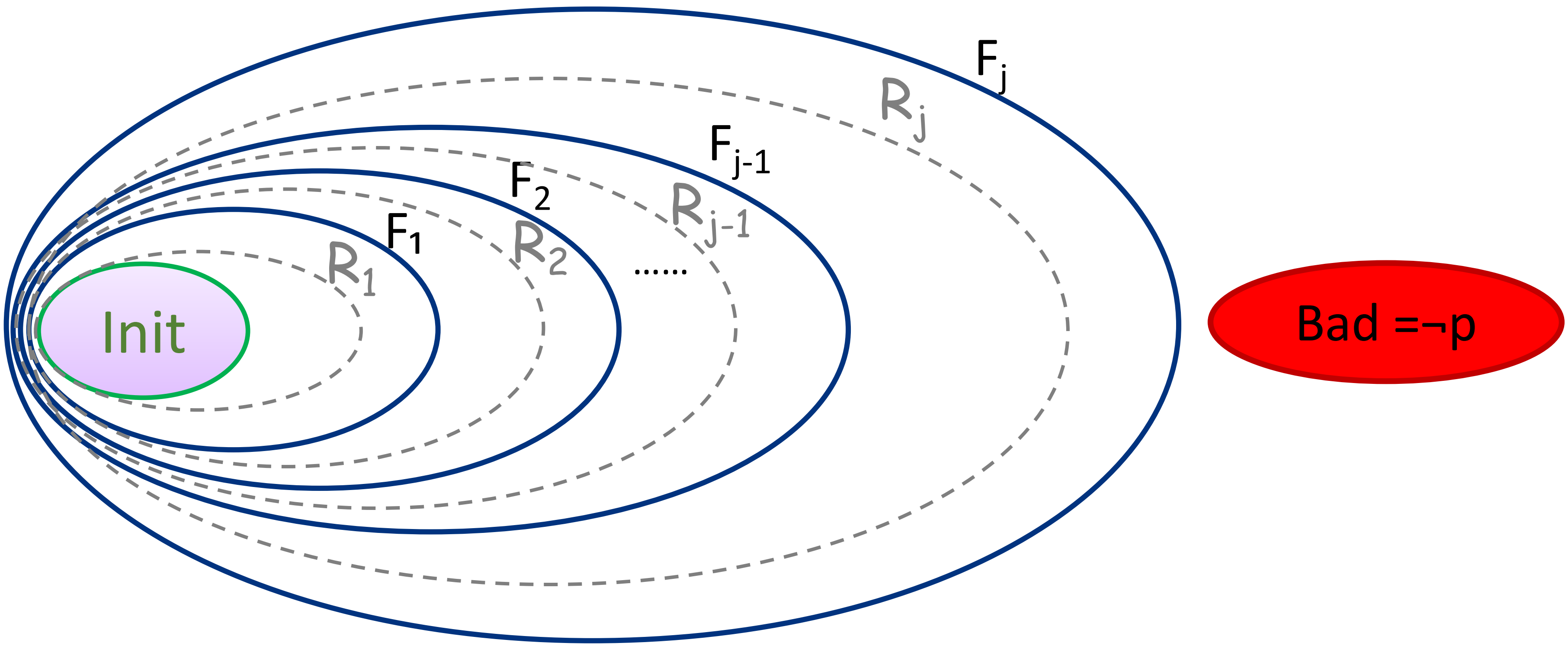
(4) $F_i \wedge TR \Rightarrow F_{i+1}'$ for $i \geq 0$ (including $F_\infty \wedge Tr \Rightarrow F_\infty'$). *(inductive)*

Remarks:
This definition is slightly different from the original definition:

- the sequence $F_0, F_1, F_2, \ldots$ is conceptually *infinite* (with $F_i = T$ for all sufficiently large i)
- we add F_∞ as the last element of the trace (as suggested in PDR)

Each F_i over-approximates states that are reachable in i steps or less (in particular, F_∞ contains all reachable states)

Safe Monotone Inductive Trace



- F_i over-approximates the states that are reachable in at most i steps
- If $F_{j+1} \Rightarrow F_j$ then F_j is an inductive invariant

Proof Obligations in IC3

A *proof obligation* in IC3 is a pair (s, i), where

- s is a (generalized) cube over state variables
- i is a natural number (called *level*)

We say that (s, i) is *blocked* (or that *s is blocked at level i*) if $F_i \Rightarrow \neg s$.
Given a proof obligation (s, i), IC3 attempts to *strengthen* the inductive trace in order to block it.

Remarks:
In IC3, s is identified with a *counterexample-to-induction* (*CTI*)

If (s, i) is a proof obligation and $i \geq 1$, then (s, i-1) is already blocked

All proof obligations are managed via a *priority queue*:

- Proof obligations with smallest level are considered first
- (additional criteria for tie-breaking)

Recursive Blocking Stage in IC3

```
// Find a counterexample, or strengthen the inductive trace
// s.t. F_N ⇒ ¬s holds
IC3_recBlockCube(s, N)
    Add(Q, (s, N))
    while ¬Empty(Q) do
        (s, k) ← Pop(Q)
        if (k = 0) return "Counterexample"
        if (F_k ⇒ ¬s) continue
        if (F_k-1 ∧ Tr ∧ s') is SAT
            t ← generalized predecessor of s
            Add(Q, (t, k-1))
            Add(Q, (s, k))
        else
            ¬t ← generalize ¬s by inductive generalization (to
                                                        level m≥k)
            add ¬t to F_m
            if (m<N) Add(Q, (s, m+1))
```

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Pushing stage in IC3

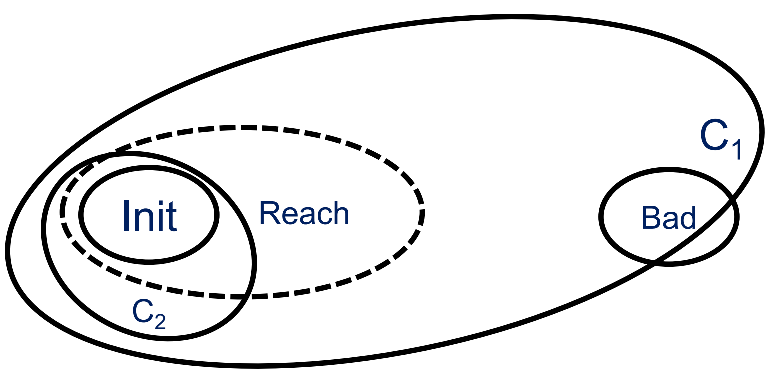
```
// Push each clause to the highest possible frame up to N
IC3_Push()
    for k = 1 .. N-1 do
        for c ∈ F_k \ F_{k+1} do
            if (F_k ∧ Tr ⇒ c')
                add c to F_{k+1}
        if (F_k = F_{k+1})
            return "Proof" // F_k is a safe inductive invariant
```

Towards improving IC3 (1)

IC3 is an excellent algorithm! So, what do we want?

We want *more control* on which lemmas to learn:

- Each lemma in the inductive trace is neither an over-approximation nor an under-approximations of reachable states (a lemma in F_k only over-approximates states reachable within k steps):
    - IC3 may learn lemmas that are *too weak* (ex. C_1) – prune less states
    - IC3 may learn lemmas that are *too strong* (ex. C_2) – cannot be in the inductive invariant



Towards improving IC3 (2)

We want to know if *an already existing lemma* is *good* (in F_∞) or *bad* (e.g., C_2 from before):

- Avoid periodically pushing bad lemmas
- Ideally, we also want to prune less useful lemmas

We want to *prioritize reusing already discovered lemmas* over learning of new ones:

- When the same cube s is blocked at different levels, usually different lemmas are discovered
  - Although, IC3 partially addresses this using pushing (and other optimizations)
- Use the same lemma to block s (at the expense of deriving additional supporting lemmas)
  - Although, in general different lemmas are of different “quality” and having some choice may be beneficial

Immediate improvement: unlimited pushing

```
// Push each clause to the highest possible frame ~~up to N~~
IC3_Push_Unlimited()
    for k = 1 .. do
        for c ∈ F_k \ F_k+1 do
            if (F_k ∧ Tr ⇒ c')
                add c to F_k+1
        if (F_k = F_k+1)
            F_∞ ← F_k
        if (F_∞ ⇒ ¬Bad)
            return "Proof" // F_∞ is a safe inductive invariant
```

Claim: after pushing F_∞ represents a *maximal inductive subset* of all lemmas discovered so far

Remark: the idea to compute maximal inductive invariants is suggested in PDR but claimed to be ineffective. In our implementation, "unlimited pushing" leads to ~10% overall speed up.

Pushing is Useful

Why pushing is useful:

- During the execution of IC3, the sets F_i are incrementally strengthened, and so it may happen that $F_k \wedge TR \Rightarrow c'$, even though this was not true at the time that c was discovered

Why pushing is good:

- By pushing c from F_k to F_{k+1}, we make F_k *more inductive* (and if F_k becomes equal to F_{k+1}, then F_k becomes an inductive invariant)
- Suppose that $c \in F_k$ blocks a proof obligation (s, k). By pushing c from F_k to F_{k+1}, we also block the proof obligation (s, k+1)

- Pushing Clauses = Improving Convergence = Reusing old lemmas for blocking bad states

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What Happens when Pushing Fails

Why pushing may fail: suppose that $c \in F_k \setminus F_{k+1}$ but $F_k \wedge TR$ does not imply c'. *Why?*

There are two alternatives:

1. c is a valid over-approximation of states reachable within $k+1$ steps, but F_k is not strong enough to imply this
   - We can strengthen the inductive trace so that $F_k \wedge TR \Rightarrow c'$ becomes true

2. c is **NOT** a valid over-approximation of states reachable within $k+1$ steps
   - There is a real *forward reachable* state r that is excluded by c
   - c has no chance to be in the safe inductive invariant
   - c is a *bad* lemma

A similar reasoning is used in:
Z. Hassan, A. Bradley, F. Somenzi: *Better Generalization in IC3*. FMCAD 2013

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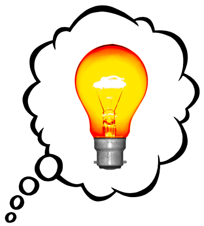
Two interdependent ideas



1. Prioritize pushing existing lemmas
   - Given a lemma $c \in F_k \setminus F_{k+1}$, we can add $(\neg c, k+1)$ as a *may-proof-obligation*
     - May-proof-obligations are “nice to block”, but do not need to be blocked
   - If $(\neg c, k+1)$ can be blocked, then c is pushed to F_{k+1}
   - If $(\neg c, k+1)$ cannot be blocked, then we discover a *concrete reachable state* r that is excluded by c and that *explains* why c cannot be inductive

2. Discover and use new forward reachable states
   - These are an *under-approximation* of forward reachable states
   - Given a reachable state, all the existing lemmas that exclude it are *bad*
     - Bad lemmas are never pushed
   - Reachable states may show that certain may-proof-obligations cannot be blocked
   - Reachable states may be used when generalizing lemmas
   - Conceptually, computing new reachable states can be thought of as *new* Init states

Quip

Input:

- A safety verification problem (Init, Tr, Bad)

Output:

- A counterexample (if the problem is UNSAFE),
- A safe inductive invariant (if the problem is SAFE)
- Resource Limit

Main Data-structures:

- A current working level N
- An *inductive trace* (same as IC3)
- A set of *proof obligations* (*similar* to IC3)
- A set R of *forward reachable states*

Proof Obligations in Quip

A proof obligation in Quip is a triple (s, i, p), where

- s is a (generalized) cube over state variables
- i is a natural number
- p $\in$ {*may*, *must*}

Remarks:

- As in IC3, if (s, i, p) is a proof obligation and $i \geq 1$, then (s, i-1) is assumed to be already blocked
- As in IC3, all proof obligations are managed via a priority queue:
  - Proof obligations with *smallest level* are considered first
  - In case of a tie, proof obligations with *smallest number of literals* are considered first
  - (additional criteria for tie-breaking)
- Have a "*parent map*" from a proof obligation to its parent proof obligation
  - parent(t) = s if (t, k-1, q) is a predecessor of (s, k, p)
  - In fact, this is usually done in IC3 as well (for trace reconstruction)

Recursive Blocking Stage in Quip (1)

1. Each time that we examine a proof obligation (s, k, p), check whether s intersects a reachable state $r \in R$

2. Discover new reachable states when possible
   - Claim: if s intersects $r \in R$ and if parent(s) exists, then there exists a reachable state r' that intersects parent(s)
     - Indeed, **ALL** states in s lead to a state in parent(s)
     - Therefore r leads to a state in parent(s) as well
   - A similar idea is present in: C. Wu, C. Wu, C. Lai, C. Huang: *A counterexample-guided interpolant generation algorithm for SAT-based model checking*. TCAD 2014

3. When (s, k, p) is blocked by an inductive lemma $\neg t$, add (t, k+1, *may*) as a new proof obligation
   - Push $\neg t$ to F_{k+1} instead of blocking (s, k+1)

4. Clear all proof obligations if their number becomes too large (important, not in pseudocode)

Recursive Blocking Stage in Quip (2)

```
// Find a reachable state r∈s, or strengthen the inductive trace
s.t. F_N ⇒ ¬s
Quip_recBlockCube(s, N, q)
    Add(Q, (s, N, q))
    while ¬Empty(Q) do
        (s, k, p) ← Pop(Q)
        if (k = 0) && (p = must) return "Counterexample"
        if (k = 0) && (p = may)
            find a state r one-step-reachable from Init,
                such that r intersects parent(s)
            add r to R; continue
        if (F_k ⇒ ¬s) continue
        if (s intersects some state r∈R) && (p = must) return
                                                  "Counterexample"
        if (s intersects some state r∈R) && (p = may)
            if parent(s) exists, find a state r' one-step-reachable
                                                         from r,
              such that r' intersects parent(s)
            add r' to R; continue
//  -- continued on the next slide --
```

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Recursive Blocking Stage in Quip (3)

```
Quip_recBlockCube(s, N, p)
//  -- continued from the previous slide --
        if (F_{k-1} ∧ Tr ∧ s') is SAT
                t ← generalized predecessor of s
                Add(Q, (t, k-1, p))
                Add(Q, (s, k, p))
        else
            ¬t ← generalize ¬s by inductive
                   generalization (to level m≥k)
            add ¬t to F_m
            if (m<N)
                if (t = s)  Add(Q, (t, m+1, p))
                else       Add(Q, (t, m+1, may))
                      // attempt to block t (not s)
```

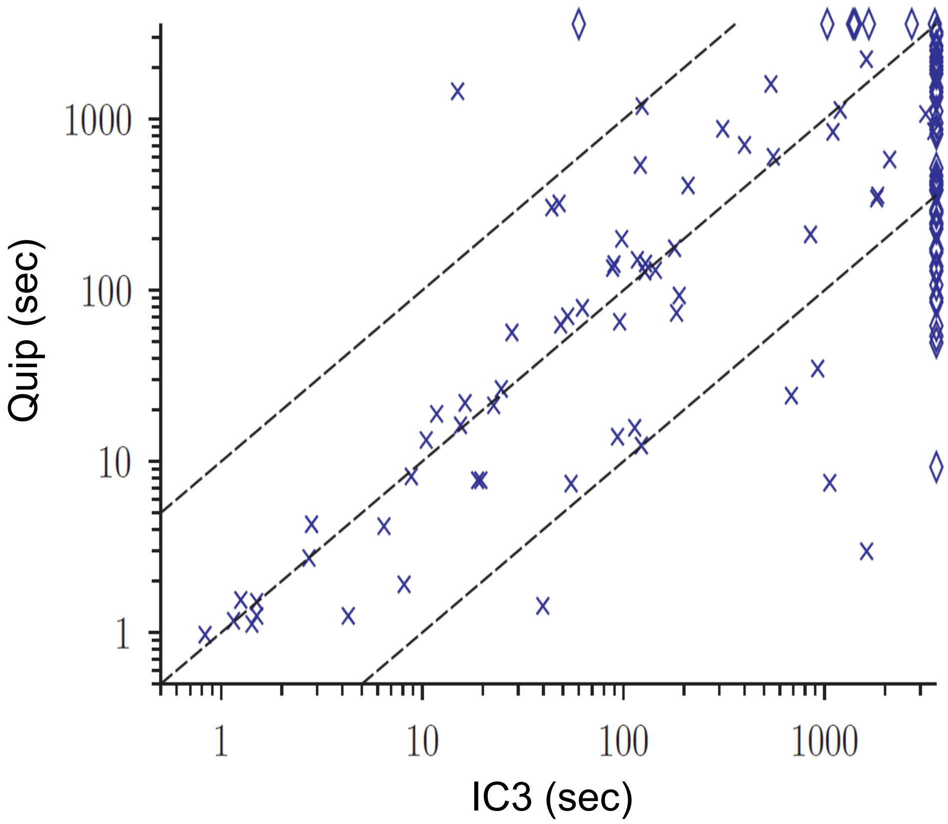
Experiments: IC3 vs. Quip on HWMCC'13 and '14

	UNSAFE solved	UNSAFE time	SAFE solved	SAFE time
`IC3`	22 (2)	52,302	76 (7)	137,244
`Quip`	32 (12)	20,302	99 (30)	69,590

Experimental results on the instances solved by either `IC3` or `Quip` separated into unsafe and safe instances. The numbers in parentheses represent the unique solves. The times are in seconds.

- Implemented in IBM formal verification tool *Rulebase-Sixthsense*
- Data for 140 instances that were not trivially solved by preprocessing but could be solved either by IC3 or Quip within 1-hour
- Detailed results at http://arieg.bitbucket.org/quip

Experiments: IC3 vs. Quip on HWMCC'13 and '14



- Data for 140 instances from prev slide

Quip – future work

- Improve handling of forward reachable states (both for performance and memory)

- Generalize forward reachable states

- Incorporate these ideas with other known IC3 developments
  - Abstraction-Refinement:
    Y. Vizel, O. Grumberg, S. Shoham: *Lazy abstraction and SAT-based reachability in hardware model checking*. FMCAD 2012
  - Lemma generalization:
    Z. Hassan, A. Bradley, F. Somenzi: *Better Generalization in IC3*. FMCAD 2013

- Experiment with other ways to combine the ideas into a full algorithm

- Lift Quip to more general domains

K-Induction without Unrolling

FMCAD'17

Arie Gurfinkel
Alexander Ivrii

K-induction Principle

[SSS2000]

Induction:

$$\frac{P(s_0) \qquad \forall i: P(s_i) \Rightarrow P(s_{i+1})}{\forall i: P(s_i)}$$

k-step Induction:

$$\frac{P(s_{0..k-1}) \qquad \forall i: P(s_{i..i+k-1}) \Rightarrow P(s_{i+k})}{\forall i: P(s_i)}$$

SAT-based Model Checking with K-induction

Let the k-unrolling of transition relation be defined as:

$U_k = T^{<0>} \wedge T^{<1>} \wedge \ldots \wedge T^{<k-1>}$

Use SAT solver to check validity of two formulas:

- Base case:

$$I^{<0>} \wedge U_{k-1} \Rightarrow P^{<0>} \ldots P^{<k-1>}$$

- Induction step:

$$U_k \wedge P^{<0>} \ldots P^{<k-1>} \Rightarrow P^{<k>}$$

If both are valid, then P is true in all the reachable states

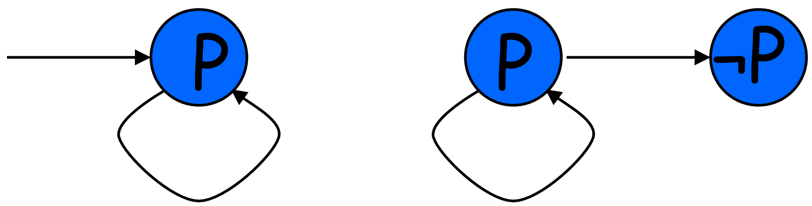
If the base case is invalid, there is a counterexample

If the induction step is invalid, increase k and try again

Simple path assumption

Unfortunately, k-induction is not complete

- some properties are not k-inductive for any k
- for example,



Simple path restriction:

- There is a path to ¬P iff there is a *simple* path to ¬P (path with no repeated states)

Complete k-induction with simple paths

Let simple($s_{0..k}$) be defined as:

- $\forall$ i,j in 0..k : (i ≠ j) $\Rightarrow s_i \neq s_j$

k-induction over simple paths:

$$\frac{P(s_{0..k-1}) \qquad \forall i: \text{simple}(s_{0..k}) \wedge P(s_{i..i+k-1}) \Rightarrow P(s_{i+k})}{\forall i: P(s_i)}$$

Must hold for k large enough, since a length of the longest simple path is bounded (recurrence diameter)

Terminology: k-invariants and k-induction

- φ is an invariant if it holds on all reachable states

- φ is a k-invariant if it holds on all states reachable in up to k steps:
  $Init(X_0) \wedge Tr(X_0, X_1) \wedge \ldots \wedge Tr(X_{N-1}, X_N) \Rightarrow \varphi(X_N)$ for all $0 \le N \le k$

- φ is a k-inductive invariant if φ is a (k-1)-invariant, and
  $\varphi(X_0) \wedge Tr(X_0, X_1) \wedge \ldots \wedge \varphi(X_{k-1}) \wedge Tr(X_{k-1}, X_k) \Rightarrow \varphi(X_k)$

- k-induction states that if φ is a k-inductive invariant, then φ is an invariant

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k-induction vs IC3

- k-induction and IC3 have complementary strengths, both theoretically and practically

- Can we understand both algorithms in a common way?
  - we present an IC3-like algorithm to show whether a given safety property is k-inductive

- Can we devise an effective algorithm combining the two?
  - we show how IC3 can be extended with k-inductive reasoning – with minor modifications of the IC3-algorithm

k-induction vs IC3

k-induction and IC3 algorithms are heavily used for unbounded model checking in both hardware and software domains

Theoretically, the two algorithms have complementary strengths:

- k-induction (with loop-free constraints) is stronger than 1-induction
- IC3 derives new lemmas to strengthen the property

Practically (on hardware benchmarks)

- k-induction is mostly successful for small values of k (up to 10)
- IC3 solves many more properties than k-induction
- However, there are properties that can be proved by k-induction with low value of k, but IC3 “gets lost”

k-induction without unrolling

- We present K-IND – an algorithm to decide whether a (k-1)-invariant formula φ is k-inductive

- K-IND returns:
  - φ is k-inductive, OR
  - φ is not k-inductive (and a counterexample to k-induction)

- Highlights:
  - Does not unroll the transition relation

  - Guarantees loop-free constraints without introducing expensive unique-state constraints

K-IND: for k=3 and without loop-free constraints

- Let φ be 2-invariant
  - No Init-state can reach a $\neg\varphi$-state in 0, 1 or 2 steps
  - Taking 0, 1 or 2 successive predecessors of a $\neg\varphi$-state cannot get to an Init-state

- We want to check/determine whether φ is 3-inductive invariant

  - $\varphi(X_0) \wedge Tr(X_0, X_1) \wedge \varphi(X_1) \wedge Tr(X_1, X_2) \wedge \varphi(X_2) \wedge Tr(X_2, X_3) \Rightarrow \varphi(X_3)$?

- Equivalently, we want to check whether the following formula is satisfiable

  - $\varphi(X_0) \wedge Tr(X_0, X_1) \wedge \varphi(X_1) \wedge Tr(X_1, X_2) \wedge \varphi(X_2) \wedge Tr(X_2, X_3) \wedge \neg\varphi(X_3)$

- Equivalently, φ is a 3-inductive invariant if and only if we cannot start from a $\neg\varphi$-state and find 3 successive predecessors satisfying φ

K-IND: demonstration

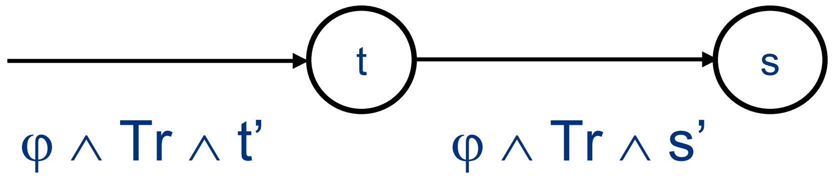
- We use an IC3-like algorithm to check satisfiability:
  $\varphi(X_0) \wedge Tr(X_0, X_1) \wedge \varphi(X_1) \wedge Tr(X_1, X_2) \wedge \varphi(X_2) \wedge Tr(X_2, X_3) \wedge \neg\varphi(X_3)$

- For simplicity, assume that $\varphi = \neg s$, where s is a cube over registers

- t is a predecessor of s
- u is a predecessor of t
- v is a predecessor of u
- We have found 3 successive predecessors: φ is not 3-inductive
  - Moreover, if v is an Init-state, then we have a counter-example to φ

K-IND: demonstration

- We use an IC3-like algorithm to check satisfiability:
  $\varphi(X_0) \wedge Tr(X_0, X_1) \wedge \varphi(X_1) \wedge Tr(X_1, X_2) \wedge \varphi(X_2) \wedge Tr(X_2, X_3) \wedge \neg\varphi(X_3)$
- For simplicity, assume that $\varphi = \neg s$, where s is a cube over registers



- Suppose now that t has no predecessors satisfying φ
- As in IC3, we can learn a lemma $\neg t$ that explains why t is not reachable
  - All SAT queries are made relative to the same frame
  - All lemmas "hold for all frames"
  - No need re-enqueue discharged proof obligations

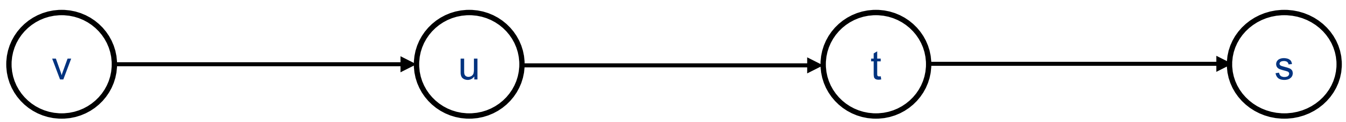
K-IND: demonstration

- Putting it all together:
  - Keep a stack of proof obligations $\{(s, i)\}$, initially $\{(\neg\varphi, 0)\}$
    - Where i represents the "depth" rather than "level"
  - Keep a set of lemmas G, initially empty
  - Iteratively:
    - Take the top proof obligation t and make a predecessor query
      $G \wedge \varphi \wedge Tr \wedge t'$
    - If found a depth-3 predecessor u and u is $Init$, return "Counter-Example"
    - Else if found a depth-3 predecessor, return "Counter-Example to 2-induction"
    - Else if found a predecessor u, add a new proof obligation $(u, i\text{-}1)$
    - Else adds $\neg t$ to G
    - If $G \Rightarrow \neg s$, return "Blocked"

- As in IC3, we can generalize $\neg t$ as long as $Init \Rightarrow \neg t$ and $G \wedge \varphi \wedge Tr \wedge \neg t \Rightarrow \neg t'$
  - in this case, the algorithm may return "Blocked" even when φ is not 3-inductive
  - But, G is still an inductive invariant proving φ

K-IND: loop-free constraints

- To integrate simple-path constraints, add (the negations of) all parent states to the predecessor queries



$\neg u \wedge \neg t \wedge \neg s \wedge \varphi \wedge Tr \wedge u'$

$\neg s \wedge \varphi \wedge Tr \wedge s'$

$\neg t \wedge \neg s \wedge \varphi \wedge Tr \wedge t'$

- Here s, t, u, v are generalized proof obligations (= sets of states)

K-IND: relatively k-inductive

- Let F be 0-invariant ($Init \Rightarrow F$)
- φ is a k-inductive invariant relative to F if φ is a k-invariant, and

$$\varphi(X_0) \wedge F(X_0) \wedge Tr(X_0, X_1) \wedge \ldots \wedge \varphi(X_{k-1}) \wedge F(X_{k-1}) \wedge Tr(X_{k-1}, X_k) \Rightarrow \varphi(X_k)$$

- K-IND can be extended to determine if φ is k-inductive relative to F

K-IND: experimental results

In practice (on hardware benchmarks, and without loop-free constraints):

- K-IND solves a few more properties than k-induction
  - Due to lemma generalization

- When solved by both, k-induction is usually faster than K-IND
  - Exactly in the same way as BMC is usually faster than IC3 when looking for counterexamples

KIC3: K-Inductive IC3

- Related Work: PD-KIND
  - “Property-Directed k-Induction”, Dejan Jovanović and Bruno Dutertre, FMCAD’2016
  - Variant of IC3/PDR based on k-induction
  - Effective on SMT benchmarks
  - Requires unrolling transition relation for validating k-inductive queries
    - A direct implementation of PD-KIND for hardware does not scale
  - Has strongly inspired our solution

- We present KIC3
  - A framework extending IC3 with k-inductive reasoning
  - Integrates k-induction into IC3 with minor modifications of the IC3-framework
  - But does not fully incorporate loop-free constraints

k-inductive blocking

- Given a proof obligation (s, i), we can attempt to block s using k-induction relative to F_{i-1}
  - Can choose any $k \leq i$

- Let's suppose that $F_{i-1} \wedge \neg s \wedge Tr \wedge s'$ is satisfiable, and t is a predecessor of s in F_{i-1}

- IC3:
  - Adds a new proof obligation $(t, i-1)$
  - Proceeds to checking whether $F_{i-2} \wedge \neg t \wedge Tr \wedge t'$ is satisfiable

- k-inductive blocking (for $k > 1$):
  - Adds a new proof obligation (t, i)
  - Proceeds to checking whether $F_{i-1} \wedge \neg t \wedge Tr \wedge t'$ is satisfiable

Comparison of KIC3-blocking and IC3-blocking

- What happens if both $F_{i-2} \wedge \neg t \wedge Tr \wedge t'$ and $F_{i-1} \wedge \neg t \wedge Tr \wedge t'$ are UNSAT?
  - IC3 learns a lemma valid (at least) to F_{i-1}
  - k-inductive blocking learns a lemma valid (at least) to F_i
  - Lemmas learned by k-inductive blocking are in general of "higher quality"

- What happens if $F_{i-2} \wedge \neg t \wedge Tr \wedge t'$ is UNSAT, while $F_{i-1} \wedge \neg t \wedge Tr \wedge t'$ is SAT?
  - IC3 stops blocking (t, i-1) and returns to blocking (s, i)
  - k-inductive blocking continues blocking predecessors of t
  - If k-inductive blocking succeeds blocking the topmost proof-obligation (s, i)
    - It may learn more lemmas, but all of these lemmas are valid (at least) to F_i

- Recall that using k-induction to block (s, i) may return "counterexample-to-k-induction"
  - Simple solution: fall back to blocking (s, i) using IC3
  - Inspired by PD-KIND: block this counterexample-to-k-induction using IC3* and continue blocking (s, i) using k-induction
  -

*or we can again use m-inductive blocking for a suitable value of m

Alternative ways to think about KIC3-blocking

- k-inductive blocking is IC3 with a slightly different strategy for managing proof obligations
  - IC3 schedules proof obligations at the lowest level they are unknown
  - k-inductive blocking may schedule proof obligations to higher levels instead

- k-inductive blocking can be thought of as abstraction
  - Given a proof obligation (t, j), IC3 checks whether $\neg t$ is inductive relative to F_{j-1}
  - However, any abstraction of F_{j-1} can be used instead
  - For example, using only lemmas from F_{i-1} closely corresponds to KIC3

Which states to block using KIC3-blocking?

- For the experiments, we modified the procedure for recursive blocking of $(\neg Bad, i)$:
  - First, use k-inductive blocking of $(\neg Bad, i)$
  - If unsuccessful, block $(\neg Bad, i)$ as usual

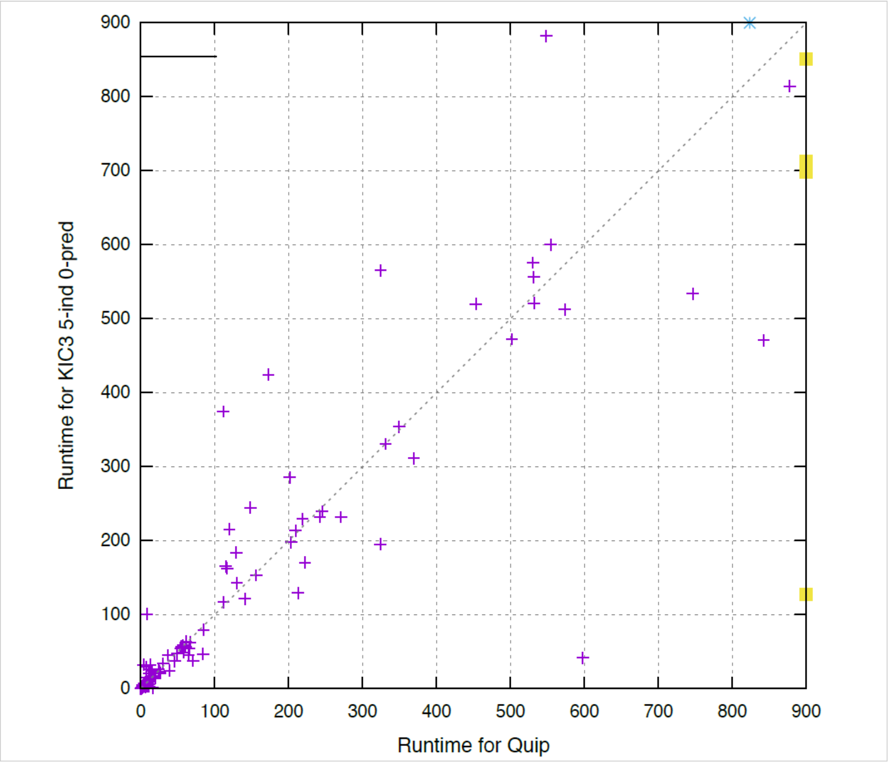
- Can also use k-inductive blocking during the pushing stage of the IC3 algorithm
  - Directly inspired by PD-KIND

Experimental Results

- Implemented in IBM's formal verification tool on top of Quip
- 238 single-property designs from HWMCC'15
  (all the designs that are not solved by simple logic synthesis, but are solved either by at least one configuration considered)
- Experimented with k = the induction depth, and m = the number of counterexamples to k-induction blocked using IC3
- 15-minutes time-limit (per property)

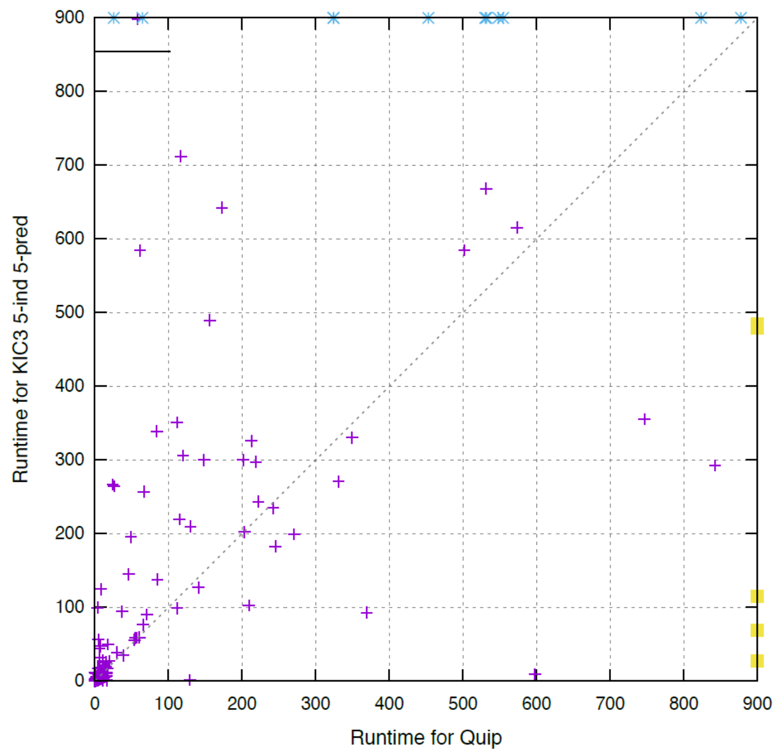
Experimental Results

- Increasing k, while fixing m=0:
  - Slightly in favor of using k-induction
  - Runtimes are highly correlated
- The scatter plot has k=5 and m=0
  - Points above diagonal = wins for IC3
  - Points below diagonal = wins for KIC3
  - IC3 solves 230 properties in 52,776 s
  - KIC3 solves 233 properties in 51,695 s
- Similar observations on proprietary designs and larger time-limits



Experimental Results

- Increasing m, while fixing k:
  - Significantly degrades performance
  - Runtimes are less correlated
- The scatter plot has k=5 and m=5
  - Points above diagonal = wins for IC3
  - Points below diagonal = wins for KIC3
  - IC3 solves 230 properties in 52,776 s
  - KIC3 solves 224 properties in 57,864 s
- Similar observations on proprietary designs and larger time-limits



Conclusion

IC3 is a great algorithm for hardware Model Checking

- but, it can still be improved

QUIP: QUest for an Inductive Proof

- aggressively push existing lemmas
- enlarge initial state by computing reachable states
- use reachable states to prune bad lemmas

KIC3: IC3 and k-induction

- k-induction without unrolling (and without simple path constraints)
- integrates easily into IC3 framework
- expensive, hard to control when to apply

IC3 is a great framework to explore MC strategies