Algorithmic Logic-Based Verification: Parameterized Systems

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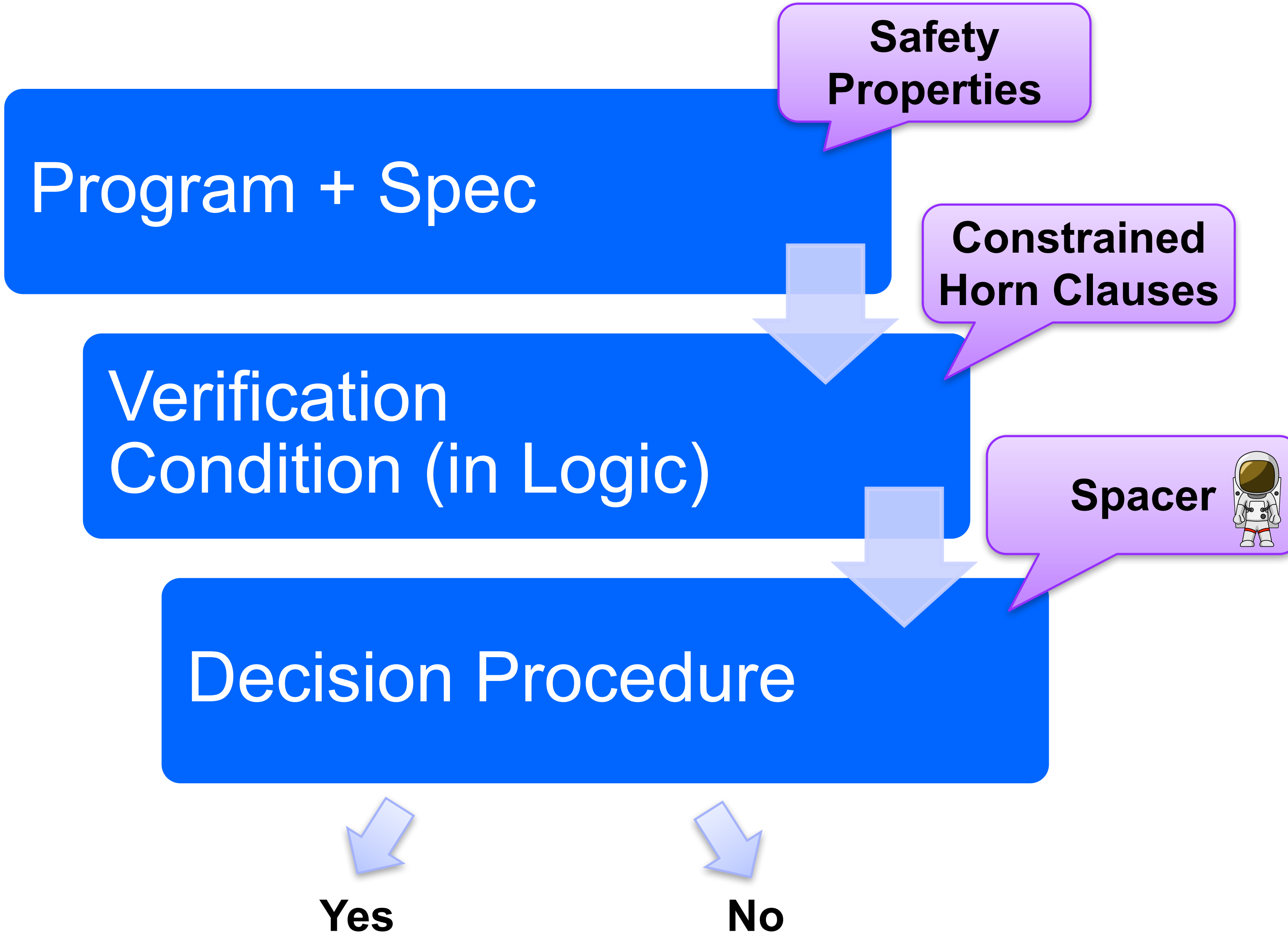
Algorithmic Logic-Based Verification



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SYMBOLIC REACHABILITY

Symbolic Reachability Problem

P = (V, Init, $\mathcal{T}r$, Bad)

P is UNSAFE if and only if there exists a number *N* s.t.

$$Init(X_0) \land \left(\bigwedge_{i=0}^{N-1} Tr(X_i, X_{i+1}) \right) \land Bad(X_N) \;\not\Rightarrow\; \bot$$

P is SAFE if and only if there exists a *safe inductive invariant Inv s.t.*

$$\left.\begin{array}{r} Init \Rightarrow Inv \\ Inv(X) \land Tr(X, X') \Rightarrow Inv(X') \end{array}\right\} \quad \boxed{\text{Inductive}}$$

$$Inv \Rightarrow \neg Bad \quad \boxed{\text{Safe}}$$

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Constrained Horn Clauses (CHC)

A Constrained Horn Clause (CHC) is a FOL formula of the forms

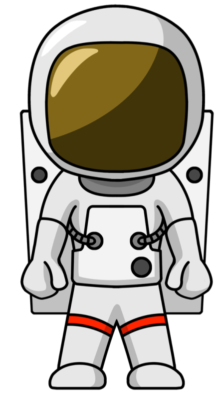
$$\forall V \,.\, (\phi \wedge p_1[X_1] \wedge \ldots \wedge p_n[X_n] \rightarrow p_{n+1}[X])$$

$$\forall V \,.\, (\phi \wedge p_1[X_1] \wedge \ldots \wedge p_n[X_n] \rightarrow \text{false})$$

where

- ϕ is a constrained in a background theory A
  - of combined theory of Linear Arithmetic, Arrays, Bit-Vectors, …
- $p_1, \ldots, p_{n+1}$ are n-ary predicates
- $p_i[X]$ is an application of a predicate to first-order terms

Spacer: Solving SMT-constrained CHC



Spacer: a solver for SMT-constrained Horn Clauses

- stand-alone implementation in a fork of Z3
- http://bitbucket.org/spacer/code

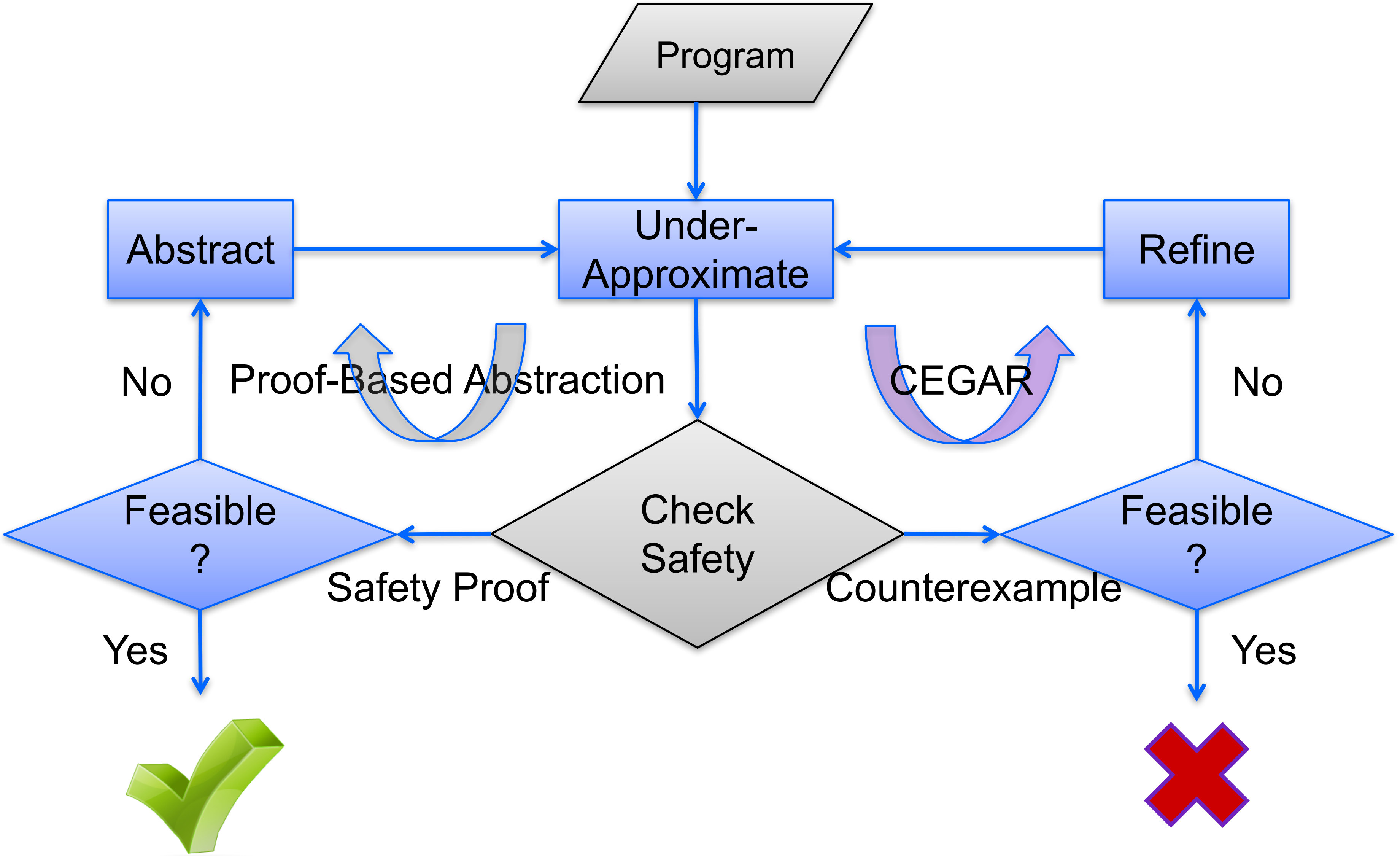
Support for Non-Linear CHC

- model procedure summaries in inter-procedural verification conditions
- model assume-guarantee reasoning
- uses MBP to under-approximate models for finite unfoldings of predicates
- uses MAX-SAT to decide on an unfolding strategy

Supported SMT-Theories

- Best-effort support for arbitrary SMT-theories
  - data-structures, bit-vectors, non-linear arithmetic
- Full support for Linear arithmetic (rational and integer)
- Quantifier-free theory of arrays
  - only quantifier free models with limited applications of array equality

Abstraction-Refinement in Spacer



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IC3, PDR, and Friends (1)

IC3: A SAT-based Hardware Model Checker

- Incremental Construction of Inductive Clauses for Indubitable Correctness
- A. Bradley: SAT-Based Model Checking without Unrolling. VMCAI 2011

PDR: Explained and extended the implementation

- Property Directed Reachability
- N. Eén, A. Mishchenko, R. K. Brayton: Efficient implementation of property directed reachability. FMCAD 2011

PDR with Predicate Abstraction (easy extension of IC3/PDR to SMT)

- A. Cimatti, A. Griggio, S. Mover, St. Tonetta: IC3 Modulo Theories via Implicit Predicate Abstraction. TACAS 2014
- J. Birgmeier, A. Bradley, G. Weissenbacher: Counterexample to Induction-Guided Abstraction-Refinement (CTIGAR). CAV 2014

IC3, PDR, and Friends (2)

GPDR: Non-Linear CHC with Arithmetic constraints

- Generalized Property Directed Reachability
- K. Hoder and N. Bjørner: Generalized Property Directed Reachability. SAT 2012

SPACER: Non-Linear CHC with Arithmetic

- fixes an incompleteness issue in GPDR and extends it with under-approximate summaries
- A. Komuravelli, A. Gurfinkel, S. Chaki: SMT-Based Model Checking for Recursive Programs. CAV 2014

PolyPDR: Convex models for Linear CHC

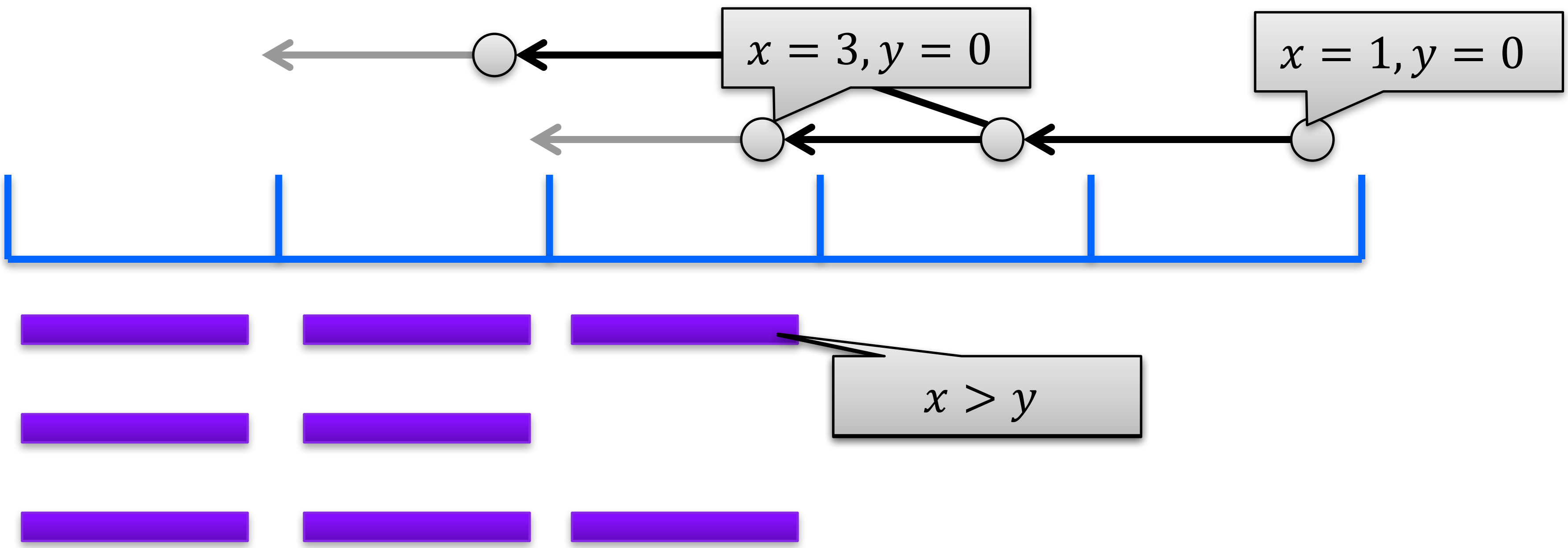
- simulating Numeric Abstract Interpretation with PDR
- N. Bjørner and A. Gurfinkel: Property Directed Polyhedral Abstraction. VMCAI 2015

ArrayPDR: CHC with constraints over Airthmetic + Arrays

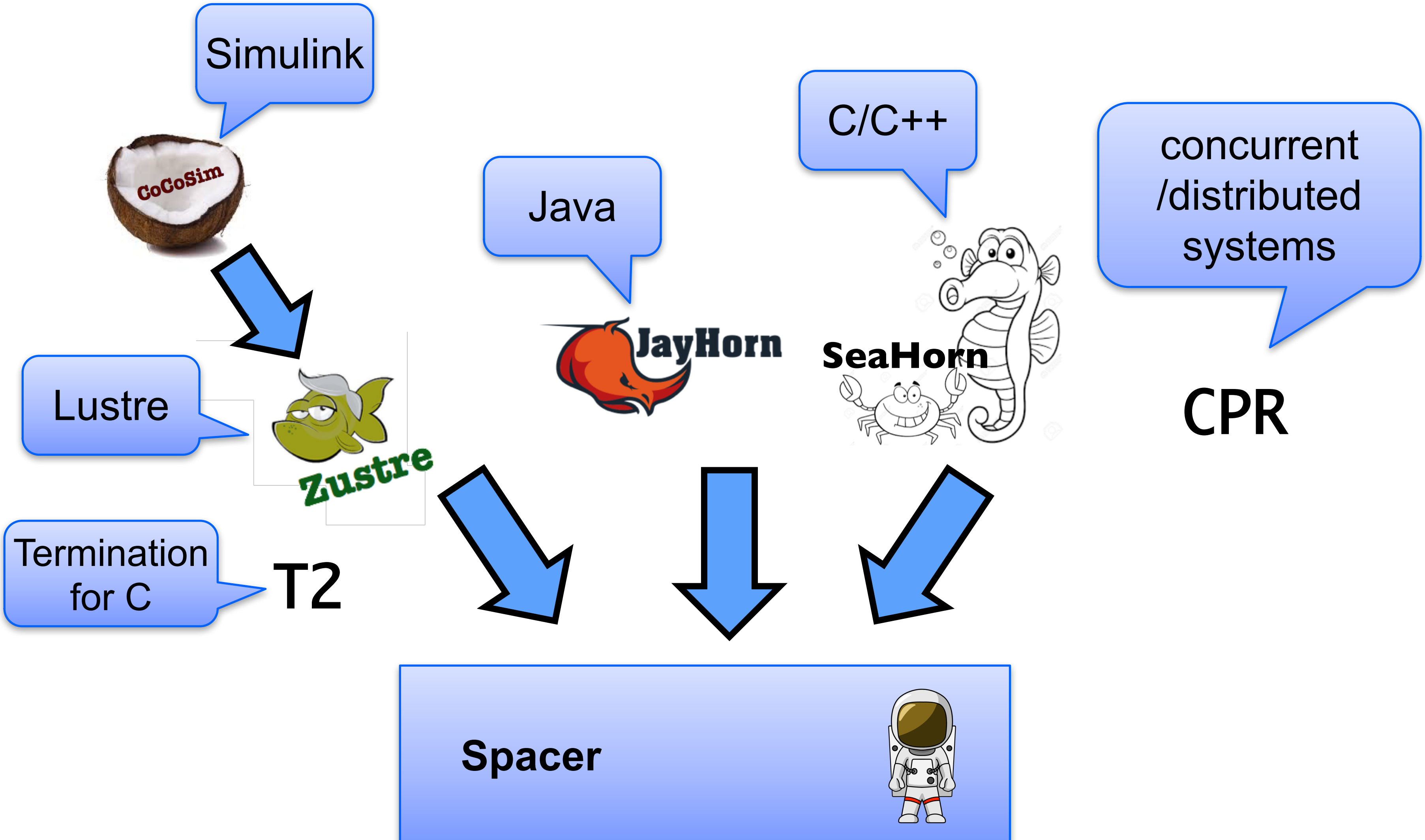
- Required to model heap manipulating programs
- A. Komuravelli, N. Bjørner, A. Gurfinkel, K. L. McMillan:Compositional Verification of Procedural Programs using Horn Clauses over Integers and Arrays. FMCAD 2015

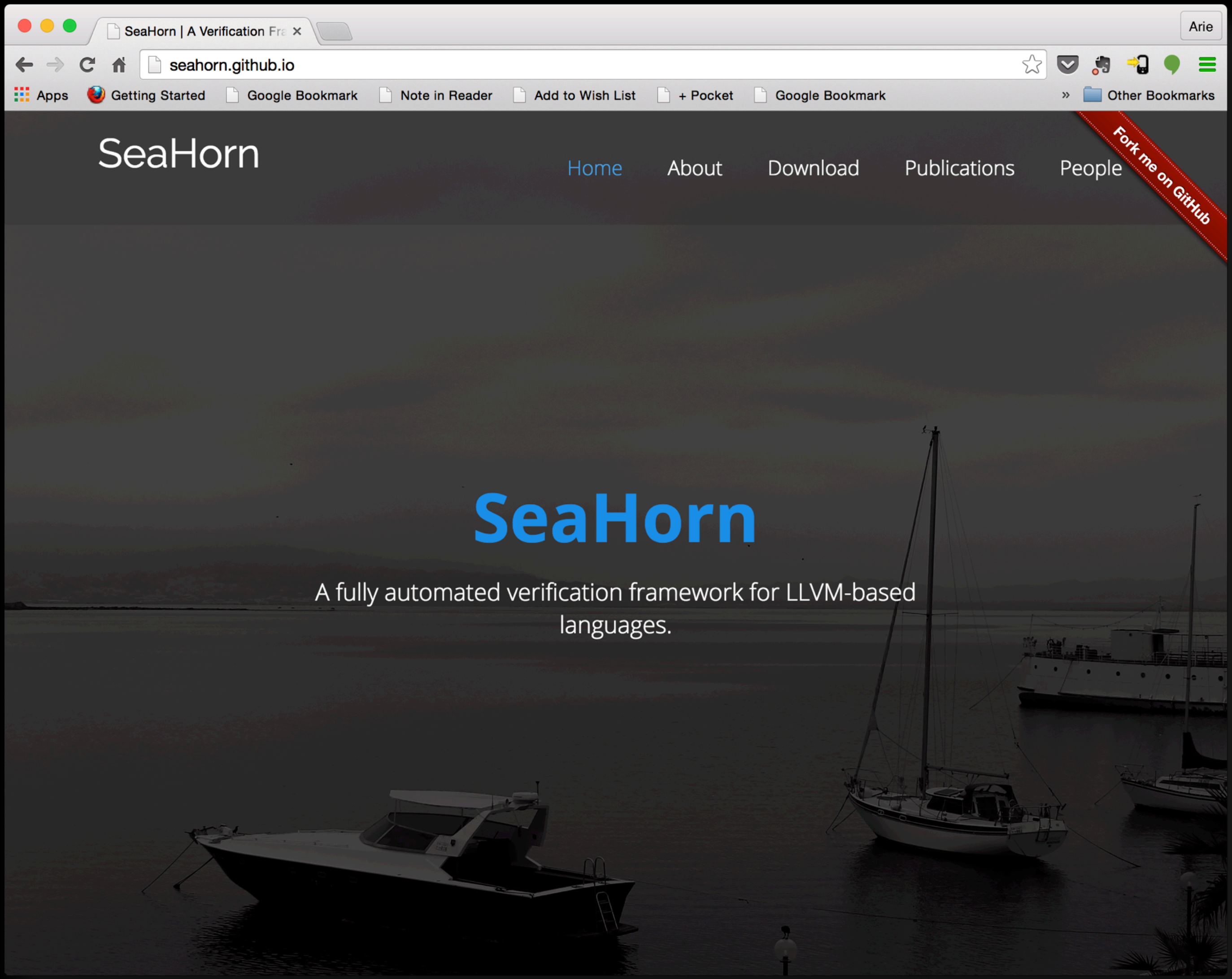
Spacer In Pictures

SpcrMkSafe



Logic-based Algorithmic Verification





http://seahorn.github.io

SeaHorn Usage

Example: in test.c, check that x is always greater than or equal to y

test.c

```c
extern int nd();
extern void __VERIFIER_error() __attribute__((noreturn));
void assert (int cond) { if (!cond) __VERIFIER_error (); }
int main(){
  int x,y;
  x=1; y=0;
  while (nd ())
  {
    x=x+y;
    y++;
  }
  assert (x>=y);
 return 0;
}
```

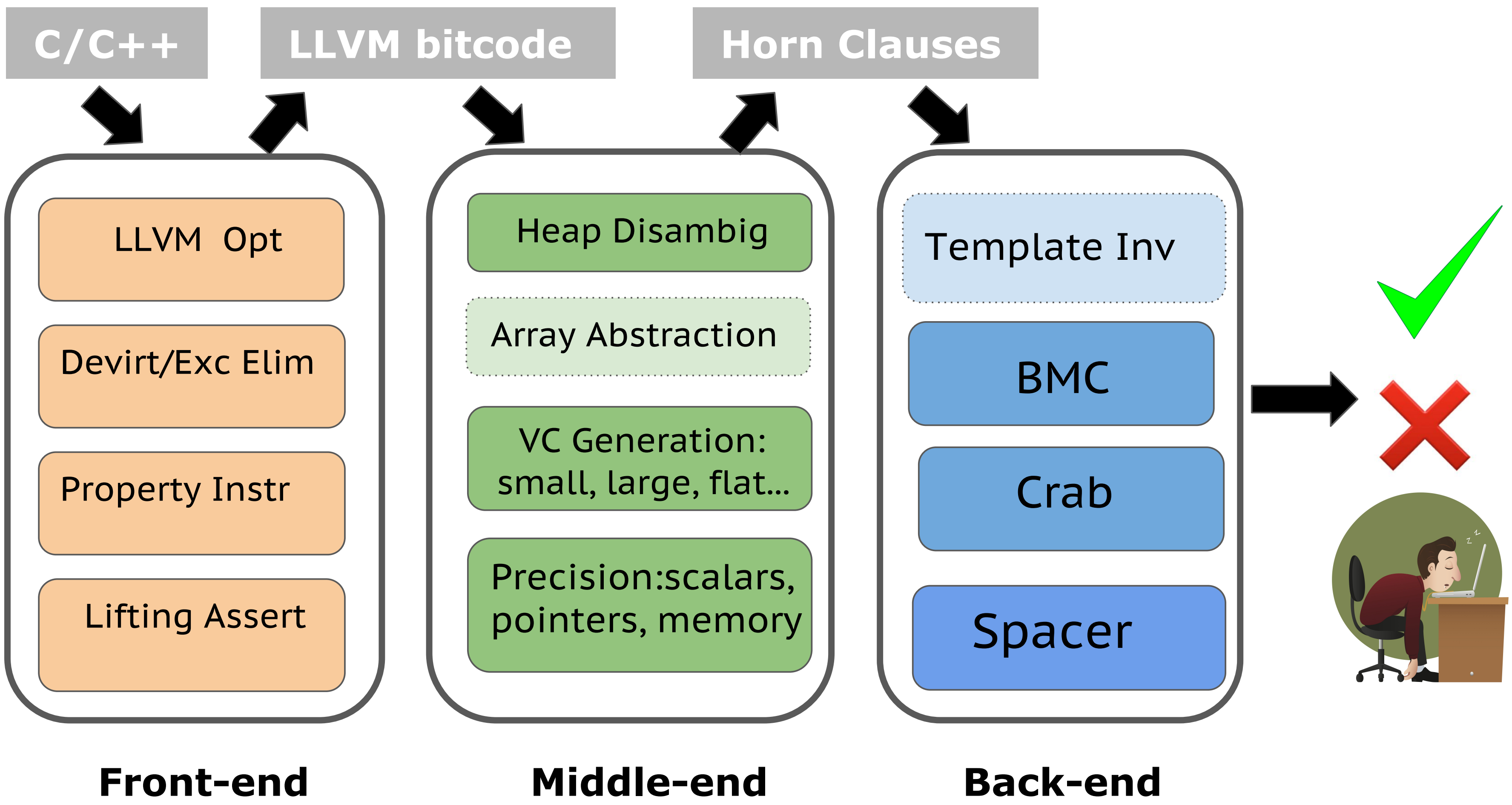
SeaHorn command:

```
)-> sea pf test.c
```

SeaHorn result:

```
          SEAHORN
----------------------------
PROPERTY (line 12) | TRUE
----------------------------
TIME(ms)           |  0.06
```

SeaHorn Architecture



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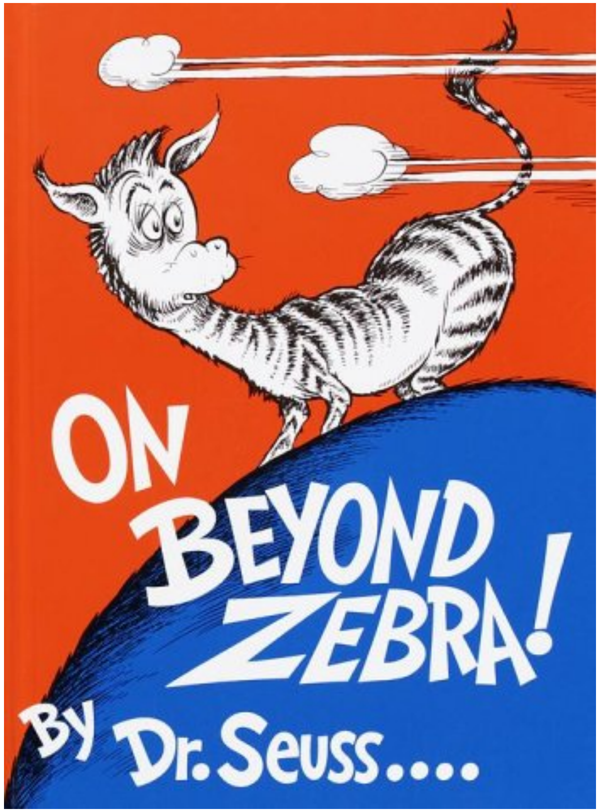

PARAMETRIZED SYMBOLIC REACHABILITY

Arie Gurfinkel, Sharon Shoham, and Yuri Meshman. SMT-Based Verification of Parameterized Systems. FSE 2016.

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What we want to do …

$$
\begin{array}{l}
\textbf{local} \\
\quad pc : \{\text{CHOOSE}, \text{TRY}, \text{WAIT}, \text{MOVE}\}\ ; \\
\quad \mathit{curr}, \mathit{next}, \mathit{desired} : \text{Location} \\
\textbf{def } \texttt{proc}(i) \textbf{ :} \\
\quad \textbf{do} \\
\qquad pc[i] = \text{CHOOSE} \ \ \textbf{:}\ \mathit{desired}[i] \leftarrow *\ ;\ pc[i] \leftarrow \text{TRY}; \\
\qquad pc[i] = \text{TRY} \wedge \forall j\,.\, i < j \Rightarrow \mathit{curr}[j] \neq \mathit{desired}[i] \wedge \mathit{next}[j] \neq \mathit{desired}[i] \\
\qquad\ \ \textbf{:} \\
\qquad\quad \mathit{next}[i] \leftarrow \mathit{desired}[i]\ ;\ pc[i] \leftarrow \text{WAIT}\ ; \\
\qquad pc[i] = \text{WAIT} \wedge \forall j\ .\ j < i \Rightarrow \mathit{next}[i] \neq \mathit{curr}[j] \wedge \mathit{next}[i] \neq \mathit{next}[j] \ \ \textbf{:} \\
\qquad\quad pc[i] \leftarrow \text{MOVE}\ ; \\
\qquad pc[i] = \text{MOVE} \textbf{ :} \\
\qquad\quad \mathit{curr}[i] \leftarrow \mathit{next}[i]\ ;\ pc[i] \leftarrow \text{CHOOSE}; \\
\textbf{def } \texttt{init}(i, j) \textbf{ :} \\
\quad pc[i] = \text{CHOOSE} \wedge \mathit{curr}[i] = \mathit{next}[i] \wedge (i \neq j \Rightarrow \mathit{curr}[i] \neq \mathit{curr}[j]) \\
\textbf{def } \texttt{bad}(i, j) \textbf{ :} \\
\quad i \neq j \wedge \mathit{curr}[i] = \mathit{curr}[j]
\end{array}
$$

Parameterized Symbolic Reachability Problem

T = (**v**, *Init*(N,**v**), *Tr*(*i*, N, **v**, **v**'), *Bad* (N,**v**))

- **v** is a set of state variables
  - each $v_k \in$ **v** is a map *Nat→Rat*
  - **v** is partitioned into *Local*(**v**) and *Global*(**v**)
- *Init*(N,**v**) and *Bad*(N,**v**) are initial and bad states, respectively
- *Tr*(*i*, N, **v**, **v**') is a transition relation, parameterized by a process identifier *i* and total number of processes N

All formulas are over the combined theories of arrays and LRA

*Init(*N*,***v***)* and *Bad(*N*,***v***)* contain at most 2 quantifiers

- Init(N,**v**) = $\forall$ x,y . φ_{Init}(N, x, y, **v**), where φ_{Init} is quantifier free (QF)
- Bad(N,**v**) = $\forall$ x,y . φ_{Bad}(N, x, y, **v**), where φ_{Bad} is QF

Tr contains at most 1 quantifier

- *Tr*(*i*, N, **v**, **v**') = $\forall$ *j* . ρ (*i*, *j*, N, **v**, **v**')

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A State of a Parameterized System

Global			
v_0	v_1	v_2	v_3

PID	Local			
	v_4	v_5	v_6	v_7
0				
1				
2				
3				
4				
5				
6				
…				
N				

Parameterized Symbolic Reachability

*T = (**v**, Init, $\mathcal{T}r$, Bad)*

T is UNSAFE if and only if there exists a number *K* s.t.

$$Init(\boldsymbol{v}_0) \wedge (\bigwedge_{s \in [0,K)} Tr(i_s, N, \boldsymbol{v}_s, \boldsymbol{v}_{s+1})) \wedge Bad(\boldsymbol{v}_K) \not\Rightarrow \bot$$

T is SAFE if and only if there exists a *safe inductive invariant Inv s.t.*

$$\left.\begin{aligned} Init(\boldsymbol{v}) &\Rightarrow Inv(\boldsymbol{v}) \\ Inv(\boldsymbol{v}) \wedge Tr(i, N, \boldsymbol{v}, \boldsymbol{v}') &\Rightarrow Inv(\boldsymbol{v}') \\ Inv(\boldsymbol{v}) &\Rightarrow \neg Bad(\boldsymbol{v}) \end{aligned}\right\} \textbf{VC(T)}$$

Parameterized vs Non-Parameterized Reachability

$$\left.\begin{aligned} Init(\boldsymbol{v}) &\Rightarrow Inv(\boldsymbol{v}) \\ Inv(\boldsymbol{v}) \wedge Tr(i, N, \boldsymbol{v}, \boldsymbol{v}') &\Rightarrow Inv(\boldsymbol{v}') \\ Inv(\boldsymbol{v}) &\Rightarrow \neg Bad(\boldsymbol{v}) \end{aligned}\right\} \textbf{VC(T)}$$

Init, Bad, and Tr might contain quantifiers

- *e.g., "ALL processes start in unique locations"*
- *e.g., "only make a step if ALL other processes are ok"*
- *e.g., "EXIST two distinct process in a critical section"*

Inv cannot be assumed to be quantifier free

- QF *Inv* is either non-parametric or trivial

Decide existence of **quantified** solution for CHC

- stratify search by the number of quantifiers
- solutions with 1 quantifier, 2 quantifiers, 3 quantifiers, etc…

ONE QUANTIFIER
TWO QUANTIFIER

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One Quantifier (Solution)

$$\left.\begin{aligned} Init(i,i,\boldsymbol{v}) &\implies Inv_1(i,\boldsymbol{v}) \\ Inv_1(i,\boldsymbol{v}) \land Tr(i,\boldsymbol{v},\boldsymbol{v}') &\implies Inv_1(i,\boldsymbol{v}') \\ j \neq i \land Inv_1(i,\boldsymbol{v}) \land Inv_1(j,\boldsymbol{v}) \land Tr(j,\boldsymbol{v},\boldsymbol{v}') &\implies Inv_1(i,\boldsymbol{v}') \\ Inv_1(i,\boldsymbol{v}) \land Inv_1(j,\boldsymbol{v}) &\implies \neg Bad(i,j,\boldsymbol{v}) \end{aligned}\right\} \textbf{VC}_{\mathbf{1}}\textbf{(T)}$$

Claim

- If VC_1(T) is QF-SAT then VC(T) is SAT
- If *Tr* does not contain functions that range over PIDs, then VC_1(T) is QF-SAT only if VC(T) admits a solution definable by a *simple* single quantifier formula
  - simple == quantified id variables do not appear as arguments to functions

VC_1(T) is essentially Owicki-Gries for 2 processes *i* and *j*

If there are no global variables then (3) is unnecessary

- VC_1(T) is linear

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How do we get it

1. Restrict Inv to a fixed number of quantifiers
   - e.g., replace Inv(N, v) with $\forall$k.Inv_1(k, N, v)

2. Case split consecution Horn clause based on the process that makes the move
   - w+1 cases for w-quantifiers
     - one for each quantified id variable
     - one for interference by “other” process (only for global variables)

3. Instantiate the universal quantifier in $\forall$k.Inv_1(k, N, v)
   - use symmetry to reduce the space of instantiations

4. Other instantiations might be needed for quantifiers if
   - id variables appear as arguments to functions

How do we get it

$$Inv(\boldsymbol{v}) \wedge Tr(j, \boldsymbol{v}, \boldsymbol{v}') \implies Inv(\boldsymbol{v}')$$

Restrict

$$(\forall k \cdot Inv_1(k, \boldsymbol{v})) \wedge Tr(j, \boldsymbol{v}, \boldsymbol{v}') \implies Inv_1(i, \boldsymbol{v}')$$

Cases

$$(\forall k \cdot Inv_1(k, \boldsymbol{v})) \wedge Tr(i, \boldsymbol{v}, \boldsymbol{v}') \implies Inv_1(i, \boldsymbol{v}')$$
$$(\forall k \cdot Inv_1(k, \boldsymbol{v})) \wedge j \neq i \wedge Tr(j, \boldsymbol{v}, \boldsymbol{v}') \implies Inv_1(i, \boldsymbol{v}')$$

Instantiate

$$Inv_1(i, \boldsymbol{v}) \wedge Tr(i, \boldsymbol{v}, \boldsymbol{v}') \implies Inv_1(i, \boldsymbol{v}')$$
$$Inv_1(i, \boldsymbol{v}) \wedge Inv_1(j, \boldsymbol{v}) \wedge j \neq i \wedge Tr(j, \boldsymbol{v}, \boldsymbol{v}') \implies Inv_1(i, \boldsymbol{v}')$$

Two Quantifier Solution

$$Init(i,j,\boldsymbol{v}) \land Init(j,i,\boldsymbol{v}) \land Init(i,i,\boldsymbol{v}) \land Init(j,j,\boldsymbol{v}) \Rightarrow I_2(i,j,\boldsymbol{v})$$
$$I_2(i,j,\boldsymbol{v}) \land Tr(i,\boldsymbol{v},\boldsymbol{v}') \Rightarrow I_2(i,j,\boldsymbol{v}')$$
$$I_2(i,j,\boldsymbol{v}) \land Tr(j,\boldsymbol{v},\boldsymbol{v}') \Rightarrow I_2(i,j,\boldsymbol{v}')$$
$$I_2(i,j,\boldsymbol{v}) \land I_2(i,z,\boldsymbol{v}) \land I_2(j,z,\boldsymbol{v}) \land Tr(z,\boldsymbol{v},\boldsymbol{v}') \land z \neq i \land z \neq j \Rightarrow I_2(i,j,\boldsymbol{v}')$$
$$I_2(i,j,\boldsymbol{v}) \Rightarrow \neg Bad(i,j,\boldsymbol{v})$$

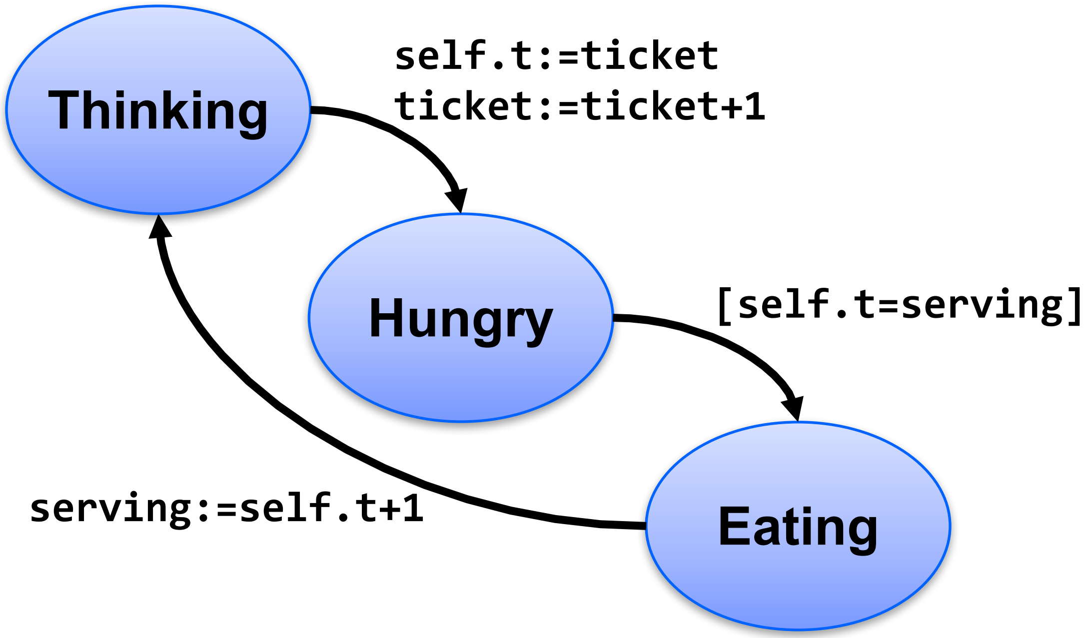
Claim

- If $VC_2(T)$ is QF-SAT then VC(T) is SAT
- If *Tr* does not contain functions that range over PIDs, then $VC_2(T)$ is QF-SAT only if VC(T) admits a solution definable by a *simple* two quantifier formula

- At least 2 quantifiers are “needed” for systems with global guards

Extends to *k*-quantifiers

Ticket Protocol



INIT: pc=Thinking, ticket=0, serving=0

BAD: self.pc=Eating, other.pc=Eating

Emacs-x86_64-10_9 Prelude - /tmp/ticket.log

```
(define-fun Inv ((id0 Int) (id1 Int) (pc (Array Int Int)) (t (Array Int Int)) (s
erving Int) (ticket Int)) Bool
 (let ((a!1 (<= (+ (select t id1) (* (- 1) (select t id0))) (- 1)))
       (a!2 (<= (+ (select t id0) (* (- 1) (select t id1))) (- 1)))
       (a!3 (or (<= (select pc id1) 1) (<= (+ serving (* (- 1) ticket)) (- 1))))
       (a!4 (or (<= (select pc id0) 0)
                (<= (+ (select t id0) (* (- 1) ticket)) (- 1))))
                                                                              )))
 (l
        (or (<= (select pc id1) 0) (<= (select pc id0) 0) a!1 a!2)
        a!3
        a!4
        a!5
        a!6
        (or a!7 (<= (select pc id1) 1) a!8 (<= (select pc id0) 0))
        a!10)))
)
```

$$\forall i, j, i \neq j \Rightarrow (i.pc \neq E) \vee (j.pc \neq E)$$

$$\forall i, i.pc = E \Rightarrow serving < ticket$$

$$\forall i, i.pc \in \{H, E\} \Rightarrow i.t < ticket$$

$$\forall i, j, i \neq j \Rightarrow (i.pc \in \{H, E\} \wedge j.pc \in \{H, E\} \Rightarrow i.t \neq j.t)$$

$$\forall i, j, i \neq j \Rightarrow (i.pc = E \wedge j.pc \in \{H, E\} \Rightarrow j.t \neq serving)$$

Putting it all together

Solve for Inductive Invariant

$k := 1$;
while *true* **do**
$\quad Inv_k(i_1, \ldots, i_k, \boldsymbol{v}) := \mathrm{Solve}(U^k(VC^\omega(T)))$;
$\quad$ **if** $Inv_k(i_1, \ldots, i_k, \boldsymbol{v}) \neq null$ **then**
$\quad\quad$ **return** *"inductive invariant found:* $\forall i_1, \ldots, i_k \,.\, Inv(i_1, \ldots, i_k, \boldsymbol{v})$*"*
$\quad res := \mathrm{ModelCheck}(T_k)$;
$\quad$ **if** $res = cex$ **then**
$\quad\quad$ **return** *"counterexample found for k processes"*
$\quad k := k + 1$

Look for bugs

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Finite vs Infinite Number of Processes

Init b[i] and move to pc=D

Move to pc=E when all distinctly init

$$
\begin{aligned}
&\textbf{def } \texttt{proc}(i) \textbf{ :}\\
&\quad \textbf{do}\\
&\qquad pc[i] = I \;:\; pc[i] := D; b[i] := 1;\\
&\qquad pc[i] = I \;:\; pc[i] := D; b[i] := 0;\\
&\qquad (\forall j \neq i \,.\, pc[i] = D \wedge pc[j] \neq I \wedge b[j] \neq b[i]) \;:\; pc[i] := E;\\
&\textbf{def } \texttt{init}(i,j) \textbf{ :}\;\; pc[i] = I;\\
&\textbf{def } \texttt{bad}(i,j) \textbf{ :}\;\; i \neq j \wedge pc[i] = E \wedge pc[j] = E;
\end{aligned}
$$

Tr does not depend on N (number of processes)

Safe for infinitely many processes. Invariant is:

$$
\begin{aligned}
Inv = (\forall i, j \,.\, i \neq j \Rightarrow (pc[i] \neq E \vee pc[j] \neq E)) \wedge\\
(\forall i \,.\, pc[i] \neq I \Rightarrow b[i] \in [0,1]) \wedge\\
(\forall i, j \,.\, (pc[i] = E \wedge i \neq j) \Rightarrow (pc[j] \neq I \wedge b[i] \neq b[j])).
\end{aligned}
$$

Unsafe for N = 2!

Evaluation and Implementation

Python-based Implementation

- Simple language for specifying concurrent protocols
- Local and Universally guarded transitions
- Constraints over arrays and integer arithmetic
- Reduce to CHC using the rules and solve using Spacer

Evaluated on Simple/Tricky Well-Know Protocols

- Dining philosophers, bakery1, bakery2, collision avoidance, ticket
- Models are pretty close to an implementation
  - limit abstraction in modeling, try to make verification hard
- Safe inductive invariants computed within seconds

Related Work

Kedar Namjoshi et al.

- Local Proofs for Global Safety Properties, and many other papers
- systematic derivation of proof rules for *concurrent* systems
- finite state and fixed number of processes

Andrey Rybalchenko et al.

- Compositional Verification of Multi-Threaded Programs, and others
- compositional proof rules for concurrent systems are CHC
- infinite state and fixed number of processes

Lenore Zuck et al.

- Invisible Invariants
- finite state and parametric number of processes
- finite model theorem for special classes of parametric systems

Nikolaj Bjørner, Kenneth L. McMillan, and Andrey Rybalchenko

- On Solving Universally Quantified Horn Clauses. SAS 2013:

Conclusion

Parameterized Verification == Quantified solutions for CHC

Quantifier instantiation to *systematically* derive proof rules for verification of safety properties of parameterized systems

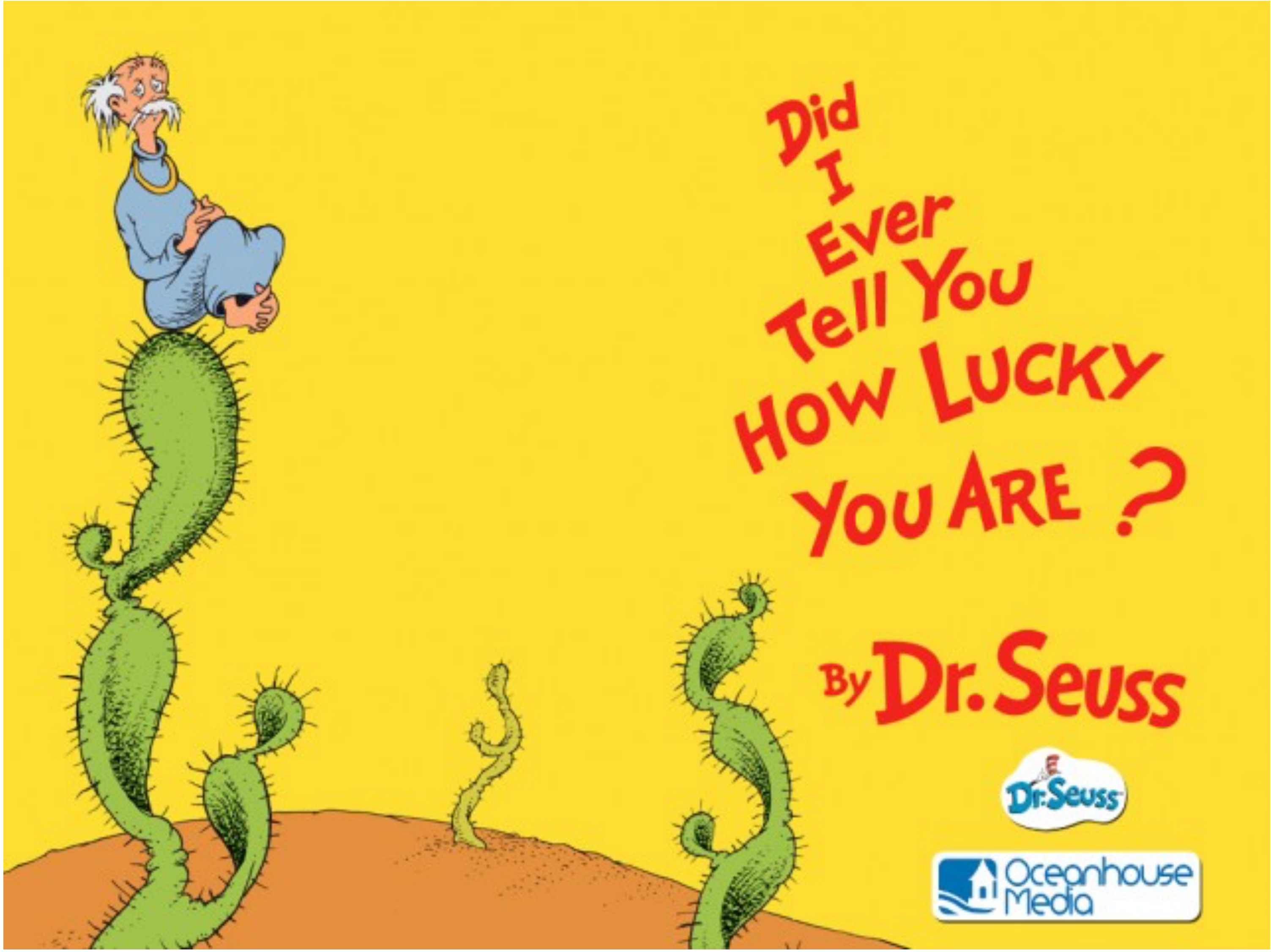
- Parameterized systems definable with SMT-LIB syntax

Lazy vs Eager Quantifier Instantiation

- eager instantiation in this talk
- would be good to extend to lazy / dynamic / model-based instantiation

Connections with other work in parameterized verification

- complete instantiation = decidability ?
- relative completeness
- …


Did
I
Ever
Tell You
How Lucky
You Are ?
By Dr. Seuss
Dr. Seuss
Oceanhouse
Media

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