BOXES: A Symbolic Abstract Domain of Boxes

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Disjunctive Refinement of an Abstract Domain

Bounded disjunctions

- extend base domain with disjunctions of size at most k
- *all* operations are done by lifting corresponding base domain operations
- easy to implement by modifying program control flow graph

Finite Powerset Domain [Bagnara et al.]

- extend base domain with all *finite* disjunctions
- *most* operations are done by lifting corresponding base domain opertions
- finding a good widening is complex (and often tricky)

Predicate Abstraction

- extend *finite* base domain with *all* disjunctions
- domain elements are represented by BDDs
- no widening required

OUR WORK

Outline

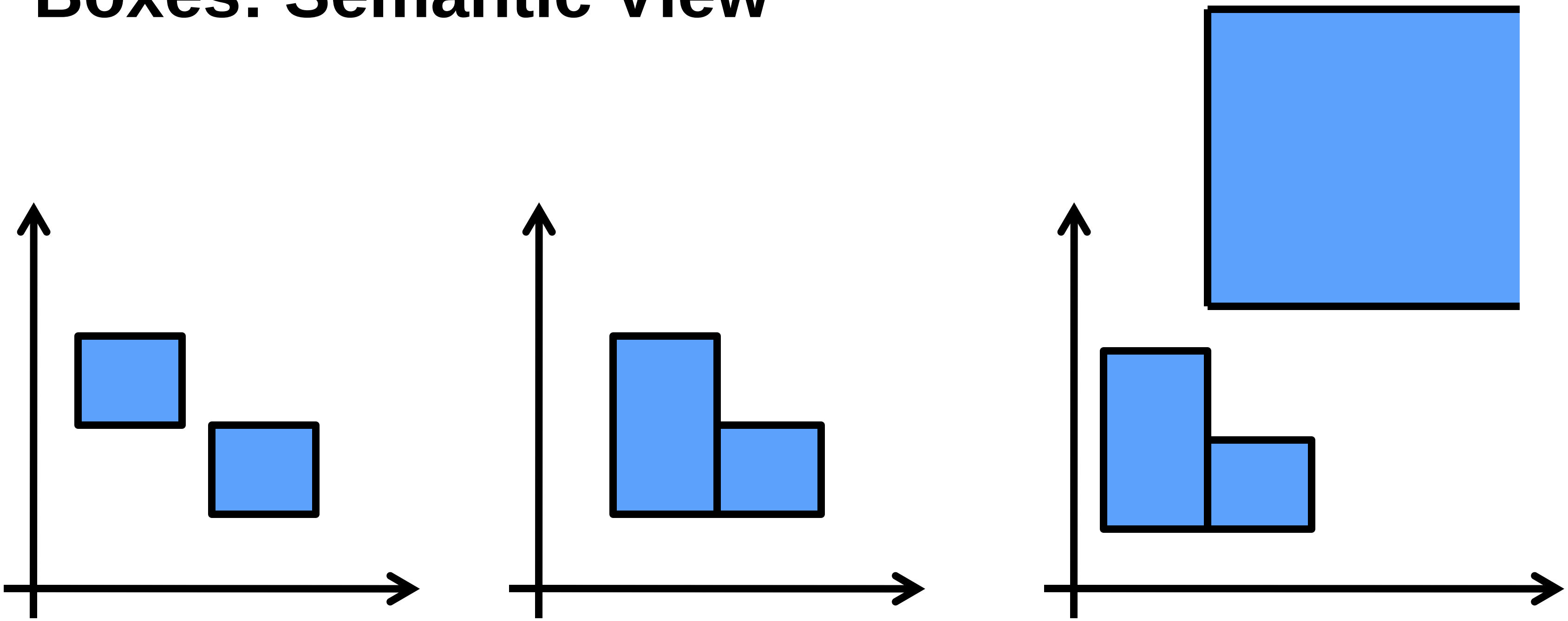
Boxes: semantics, representation, operations

Widening

Experiments

Conclusion

Boxes: Semantic View



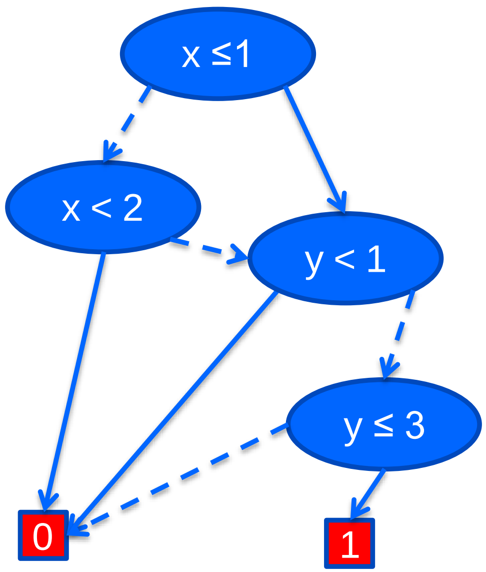
Boxes are “finite union of box values”

(alternatively)

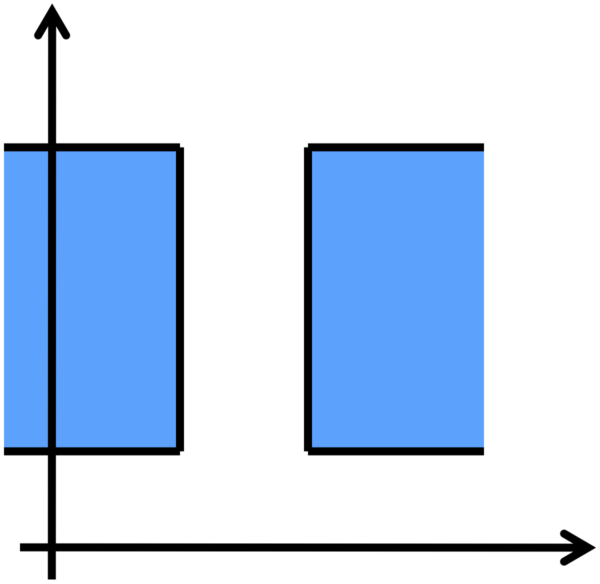
Boxes are “Boolean formulas over interval constraints”

Boxes: Representation

LDD



Semantics



Represented by (Interval) Linear Decision Diagrams (LDD)

- BDDs + non-terminal nodes are labeled by interval constraints + extra rules
- retain complexity of BDD operations
- canonical for Boxes
- available at http://lindd.sf.net

Domain Operations

Basic domain operations are ed by LDD operations

meet

order
(semantic)

join

Operation	Complexity
$f \wedge g$	O(|f||g|)
ITE(h, f, g)	O(|h||f||g|)
$\neg f$	O(1)

Operation	Complexity
$f \vee g$	O(|f||g|)
$f \Rightarrow g$	O(|f||g|)
$\exists U.\ f$	$O(\|f\|\ 2^{\|U\|})$

projection

Additional operations

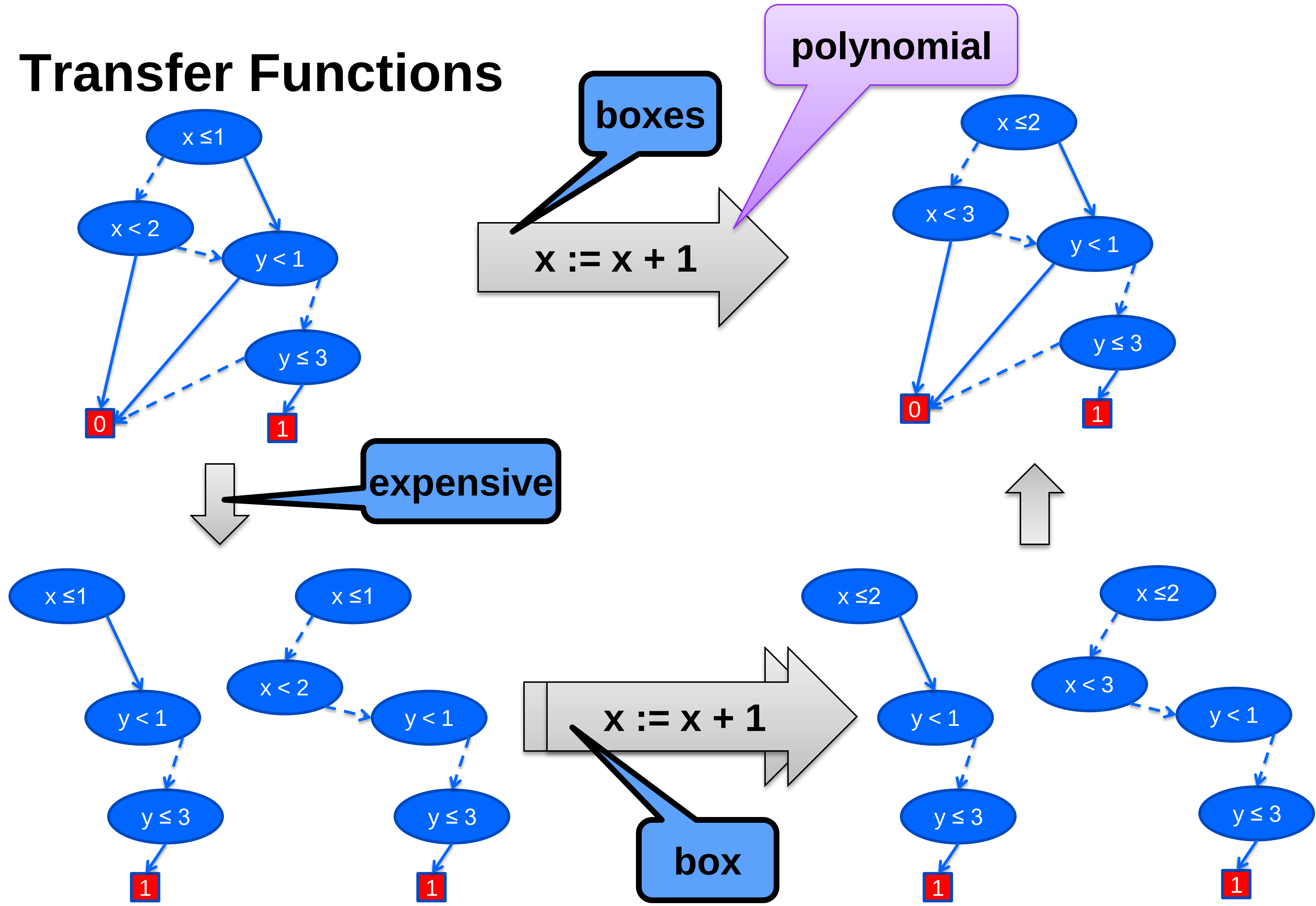
- set difference $f \setminus g$ implemented by $f \wedge \neg g$
- BoxHull (f) – smallest Box containing f
- BoxJoin (f, g) – smallest Box containing the union of Box f and Box g

used to compare

Box and Boxes

All operations are polynomial in the size of the representation

Transfer Functions



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Boxes: semantics, representation, operations

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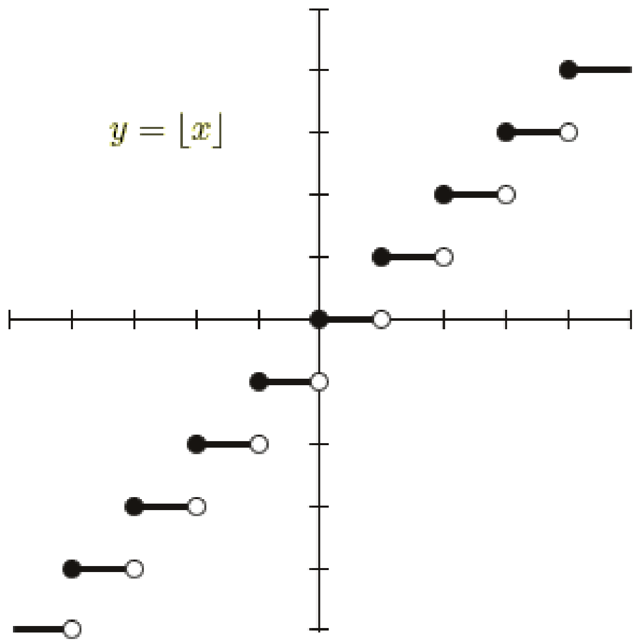
Step Function



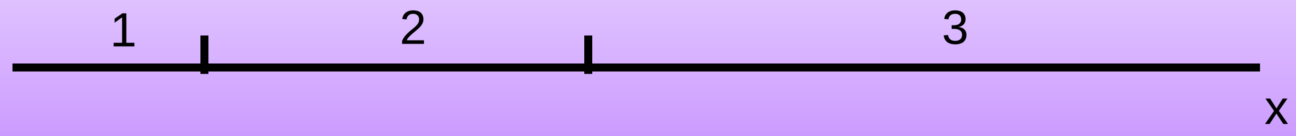
A function on the reals $\mathbb{R}$ is a *step function* if it can be written as a *finite* linear combination of semi-open intervals

$$f(x) = \alpha_1 f_1(x) + \cdots + \alpha_n f_n(x)$$

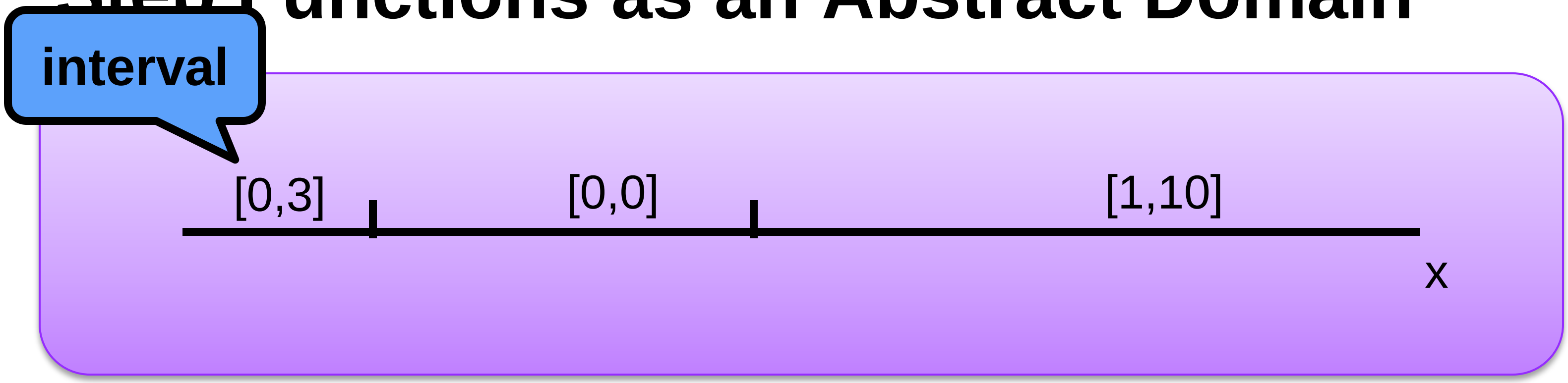
where $f_i \in \mathbb{R}$ and $\alpha_i(x)=1$ if $x \in [a_i, b_i)$ and 0 otherwise, for i=1,…,n

Weisstein, Eric W. "Step Function." From *MathWorld*--A Wolfram Web Resource.
http://mathworld.wolfram.com/StepFunction.html

Step Functions as an Abstract Domain



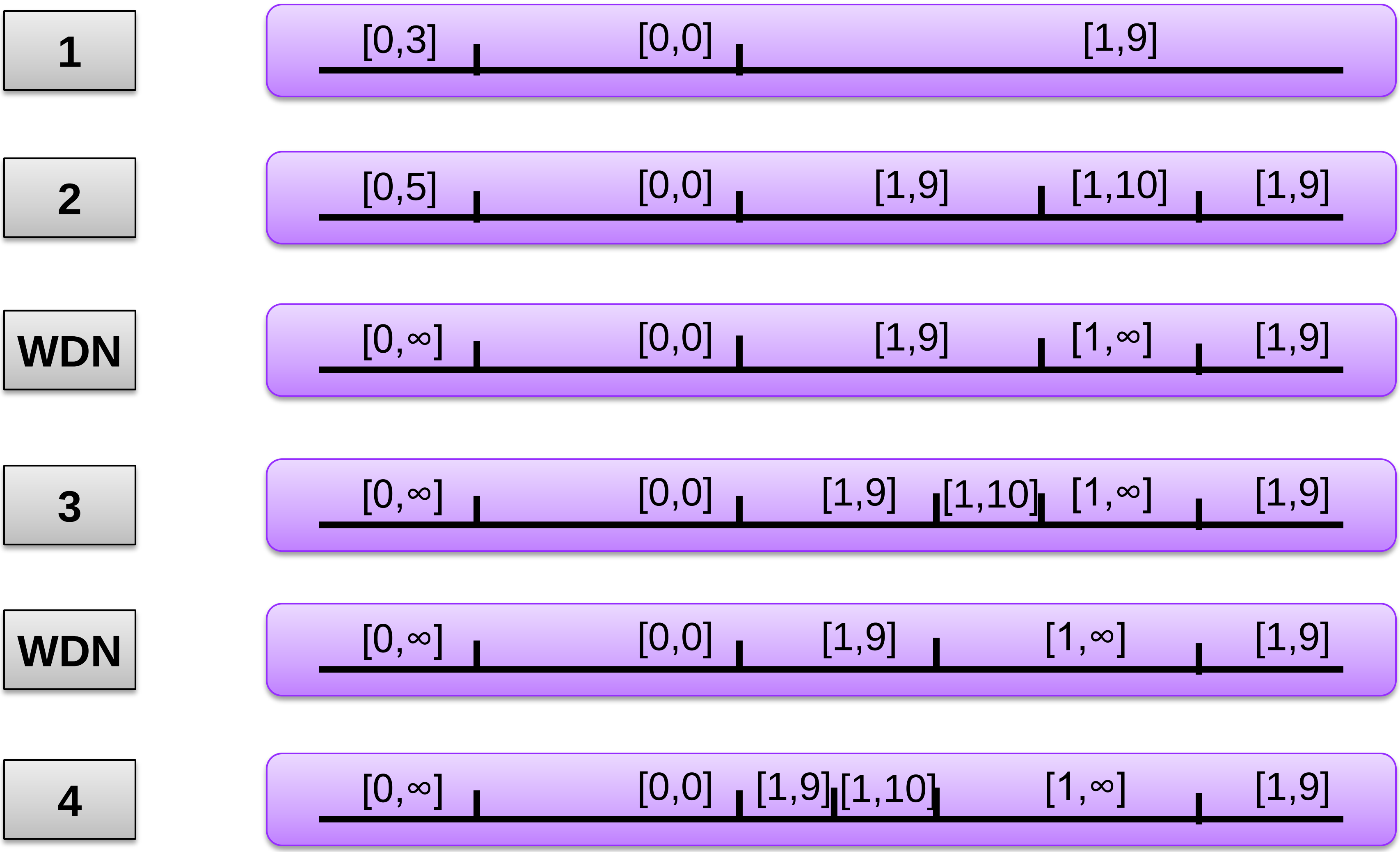
Step Functions as an Abstract Domain



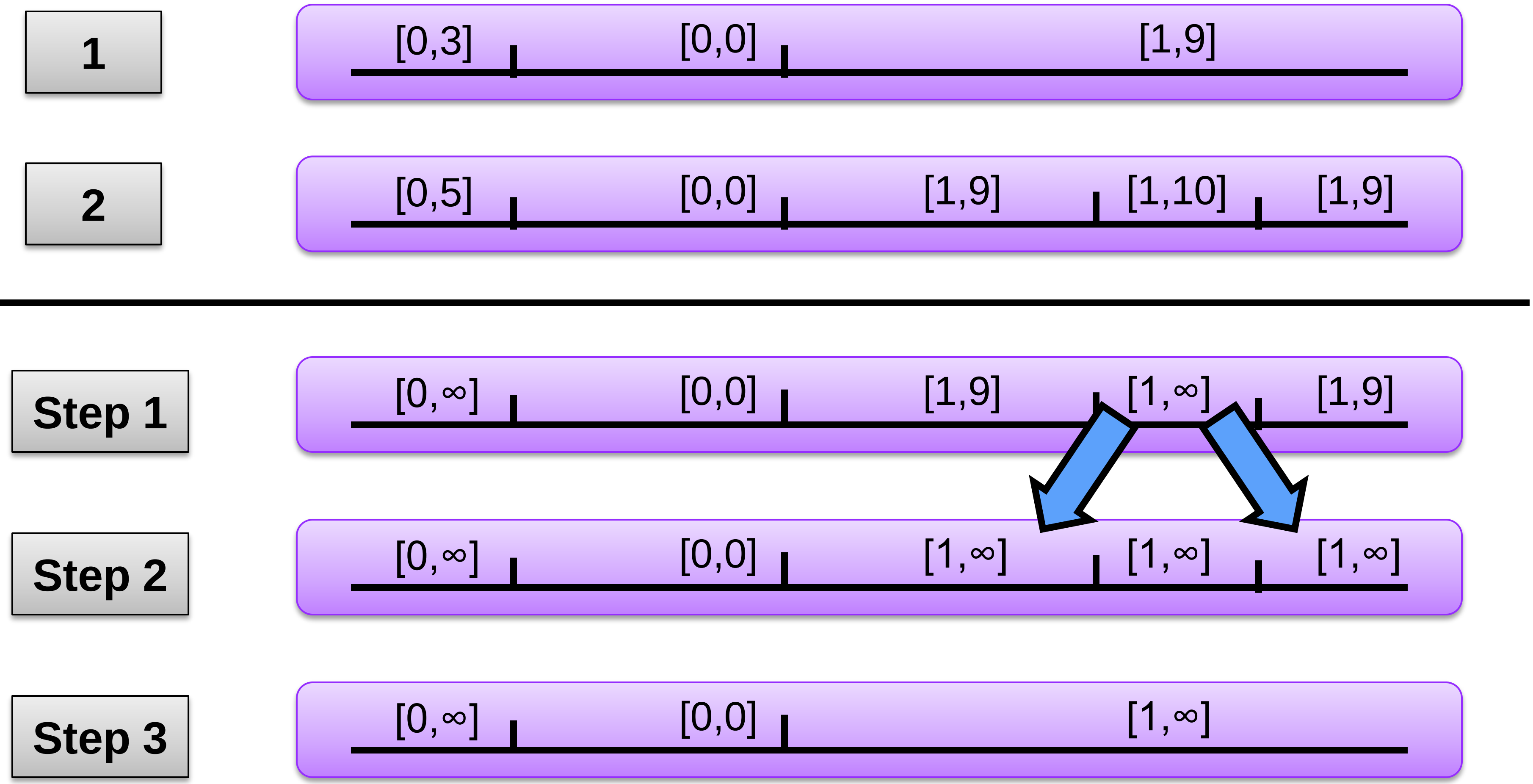
STEP(D) an abstract domain of step functions over an abstract domain D

- elements are step functions $\mathbb{R} \rightarrow D$
- order is pointwise: $f \sqsubseteq g$ iff $\forall x . f(x) \sqsubseteq_D g(x)$
- join is pointwise: $f \sqcup g$ is $\lambda x . f(x) \sqcup_D g(x)$
- meet is pointwise: $f \sqcap g$ is $\lambda x . f(x) \sqcap_D g(x)$
- widen is pointwise: $f \nabla g$ is $\lambda x . f(x) \nabla_D g(x)$????

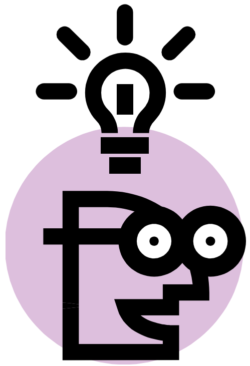
Pointwise Extension of Widen Diverges



Widening for Step Functions



Back to Boxes



Boxes are Step functions!

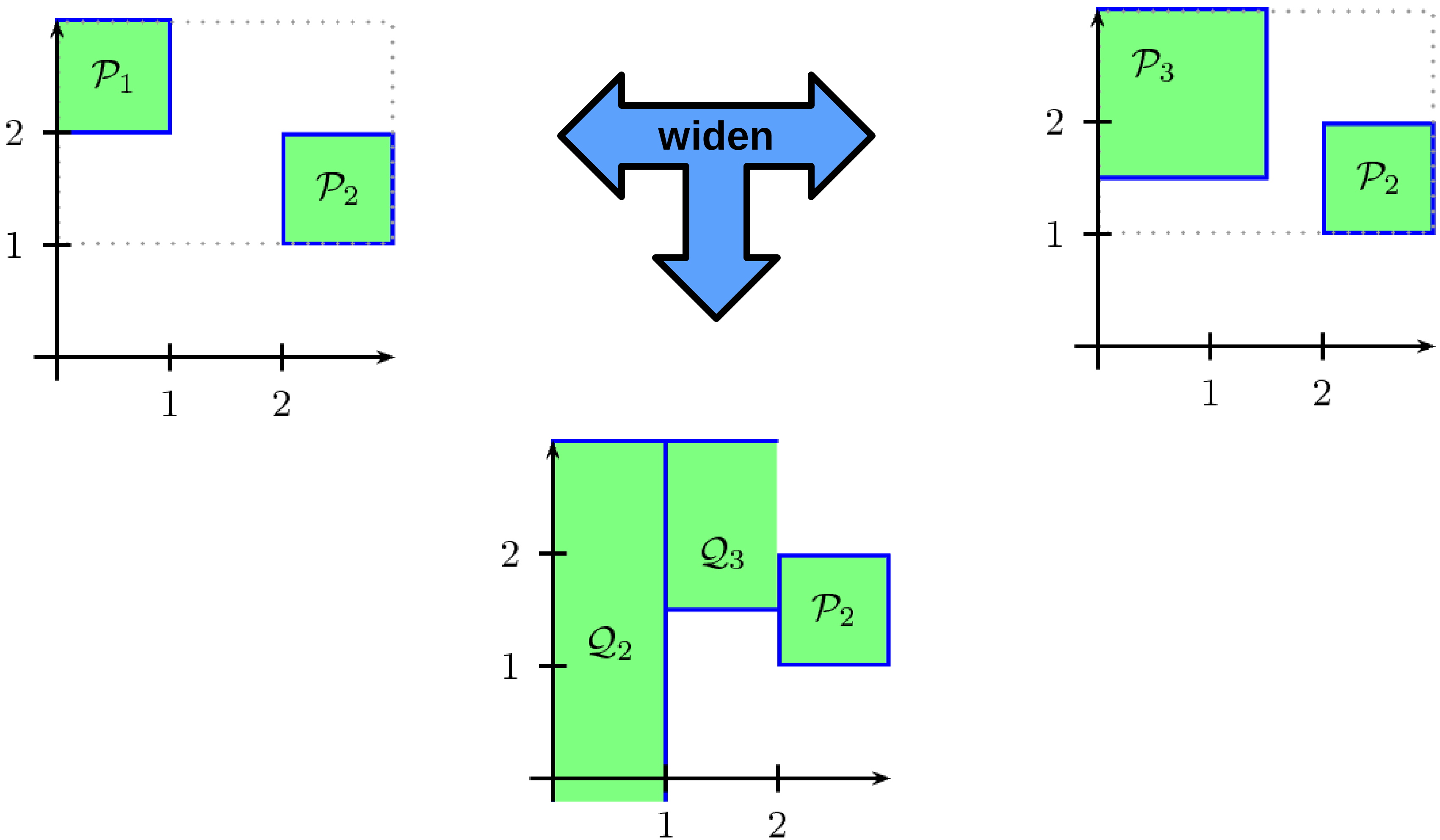
- 1-dim Boxes are STEP($\{\bot,\top\}$) $\mathbb{R}\to\{\bot,\top\}$
- 2-dim Boxes are STEP (STEP ($\{\bot,\top\}$) $\mathbb{R}\to\mathbb{R}\to\{\bot,\top\}$
- n-dim Boxes are STEP^n ($\{\bot,\top\}$) $\mathbb{R}^n\to\{\bot,\top\}$

Widen for $\{\bot,\top\}$ is trivial

Widen for n-dim Boxes is defined recursively on dimensions

In the paper, a polynomial time algorithm that implements this widen operator directly on LDDs.

Widen: An Example



Boxes versus Finite Powersets

	Boxes	Finite Powerset
Base domain	Box	Any
Representation	Decision Diagram	Set / DNF
Domain order	semantic	syntactic
Complexity	polynomial in representation	polynomial in representation
Singleton Widen	Box	base domain
Widen	Step Function	Multiple Choices

Experiments: Invariant Computation

Abstract Domains

- LDD Box – Box domain using LDDs
- LDD Boxes – Our Boxes domain using LDDs
- PPL Box – `Rational_Box` of Parma Polyhedra Library (PPL)
- PPL Boxes – `Pointset_Powerset<Rational_Box>` of PPL

Analyzer

- custom analyzer on top of LLVM compiler infrustructure
- computes loop invariants for all loops over all SSA variables in a function

Benchmark

- from open source software: mplayer, CUDD, make, …
- Stats: 5,727 functions

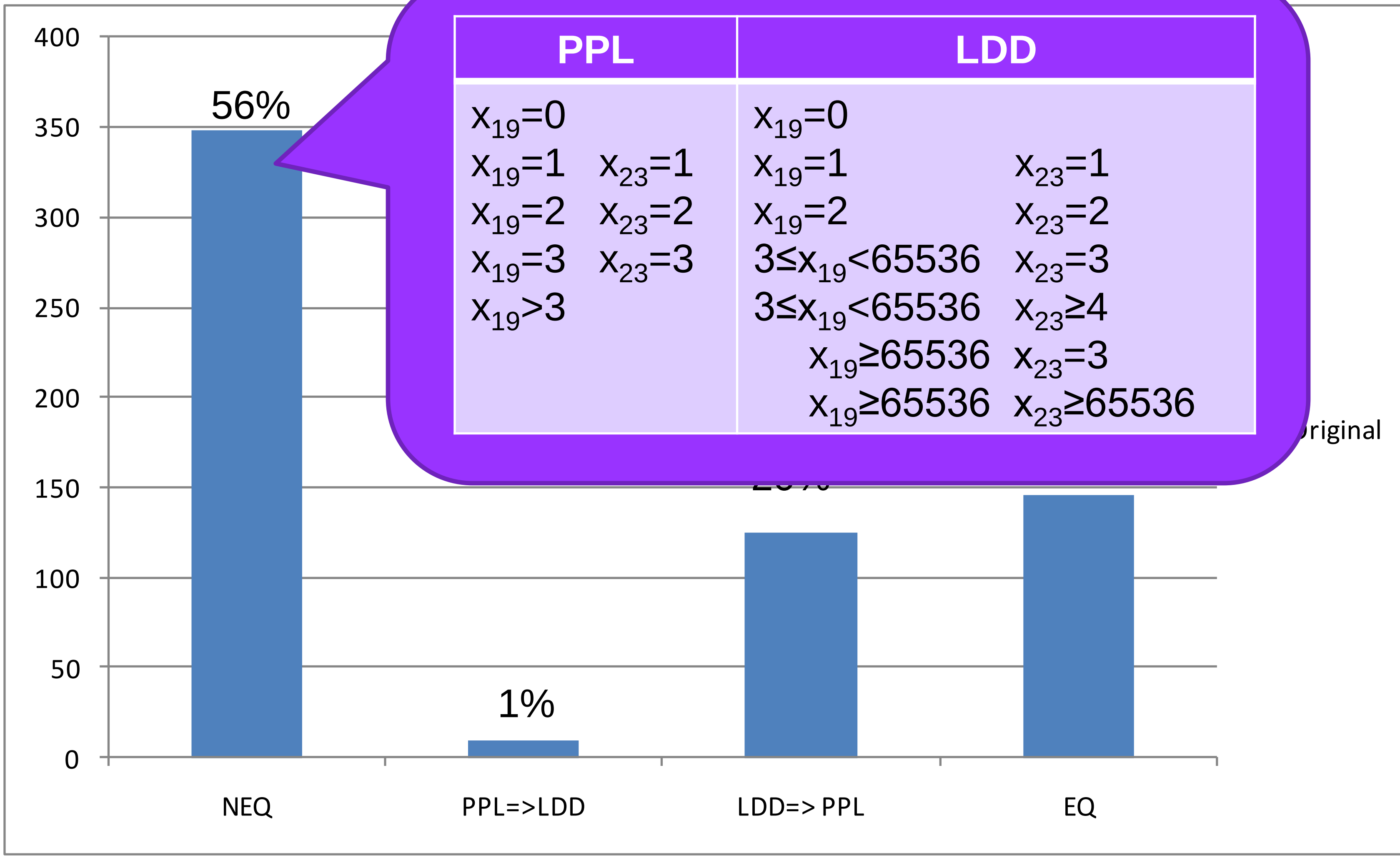
  9 – 9,052 variables (avg. 238, std. 492)

  0 – 241 loops (avg. 7, std. 12)

Results: Time

Domain	%Solved (w/ 60s TO)	Time (m)	% in Basic	% in Image	% in Widen
LDD Box	99.8%	4	77%	23%	0%
PPL Box	96.1%	117	86%	14%	0%
LDD Boxes	87.9%	118	61%	38%	1%
PPL Boxes	14.2%	201	95%	1%	3%

Results: Precision



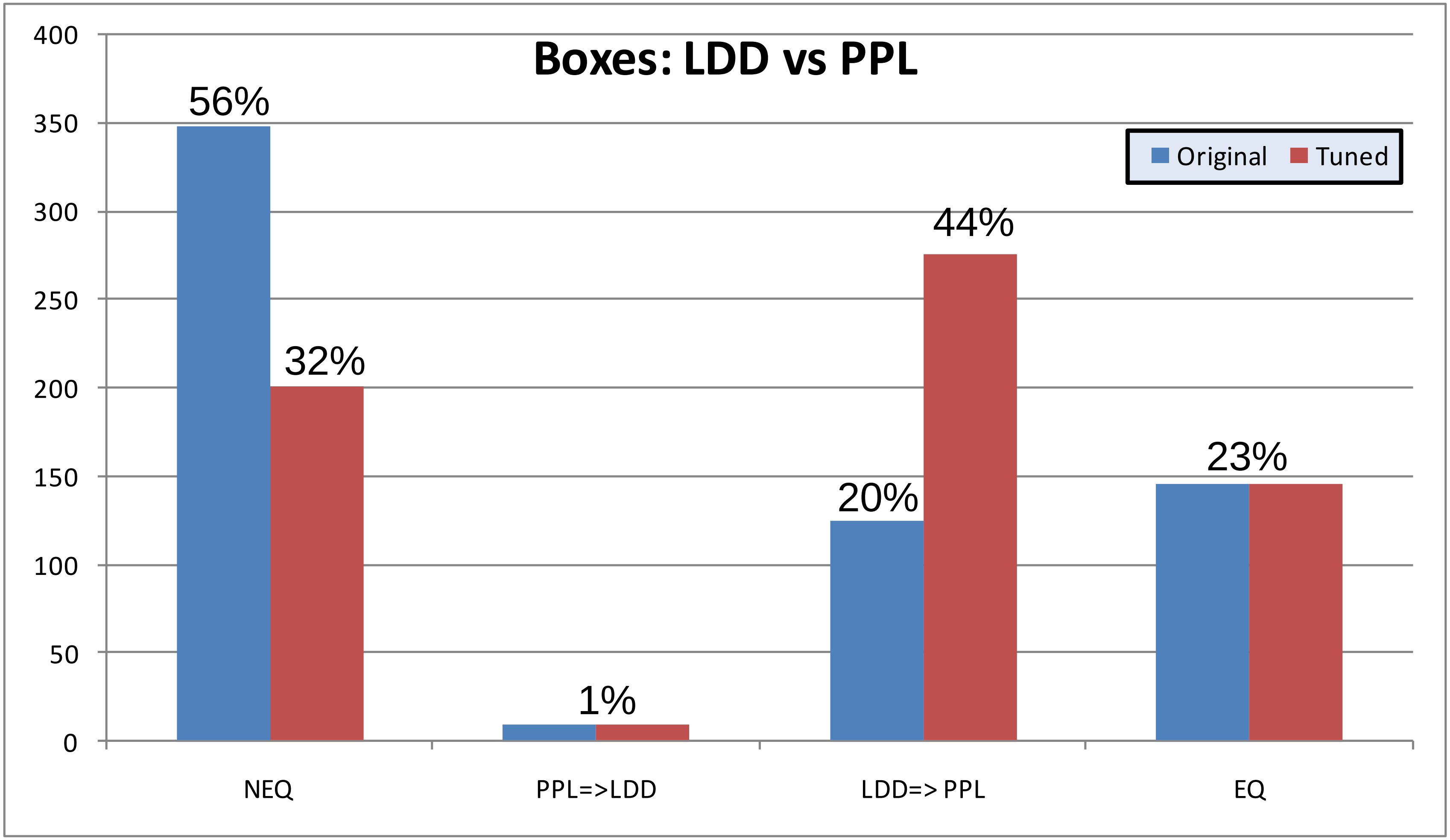
PPL		LDD	
$x_{19}=0$		$x_{19}=0$	
$x_{19}=1$	$x_{23}=1$	$x_{19}=1$	$x_{23}=1$
$x_{19}=2$	$x_{23}=2$	$x_{19}=2$	$x_{23}=2$
$x_{19}=3$	$x_{23}=3$	$3 \leq x_{19} < 65536$	$x_{23}=3$
$x_{19}>3$		$3 \leq x_{19} < 65536$	$x_{23} \geq 4$
		$x_{19} \geq 65536$	$x_{23}=3$
		$x_{19} \geq 65536$	$x_{23} \geq 65536$

Widening: PPL vs LDD

```
x = 0;
y = 0
while (1){
   x++;
   y++;
}
```

	PPL		LDD
Iteration 1	x=0 y=0	≡	x=0 y=0
Iteration 2	x=0 y=0 x=1 y=1	≡	x=0 y=0 x=1 y=1
Widen 1	x=0 y=0 x=1 y=1 1<x	⊇	x=0 y=0 1≤x y=1
Iteration 3			x=0 y=0 1≤x y=1 2≤x y=2
Widen 2			x=0 y=0 1≤x y=1 1≤x 2≤y

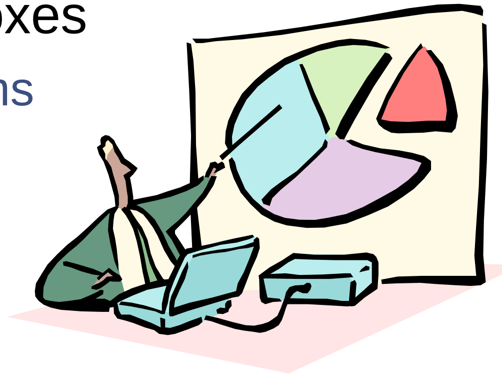
Results: Precision w/ Tuned Widening



Conclusion

Boxes: A new disjunctive abstract domain of sets of boxes

- efficient representation based on Linear Decision Diagrams
- semantic order relation
- efficient operations and widening
- more precise and efficient than finite powersets of box



A new widening scheme

- lifting widening from a base domain to the domain of step functions

Future Work

- applications
- extending the technique to richer base domains, i.e., octagons, TVPI
  - representation and base operations are easy (already exist in LDD)
  - widening?

http://lindd.sf.net

THE END

Transfer Functions: PPL vs LDD

PPL

$x=1$
$x=2$

y := x →

$x=1 \; y=1$
$x=2 \; y=2$

$\equiv$ $\qquad$ $\subseteq$

LDD

$1 \leq x \leq 2$

y := x →

$1 \leq x \leq 2, 1 \leq y \leq 2$

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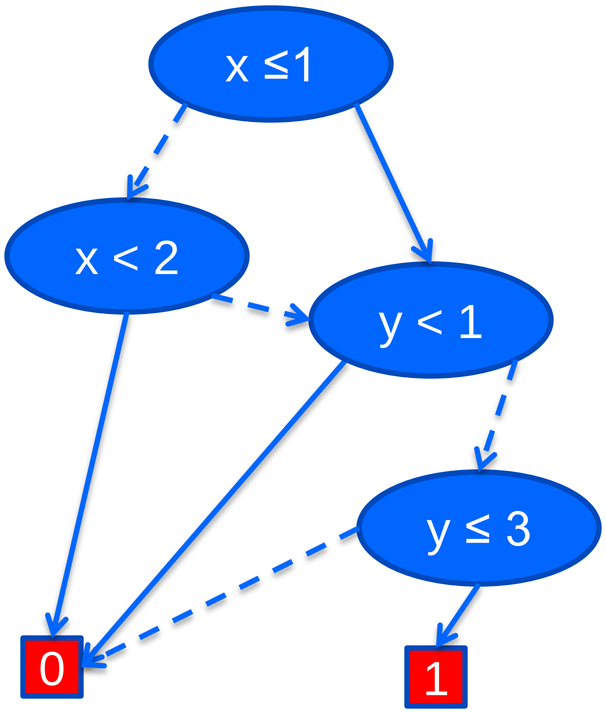
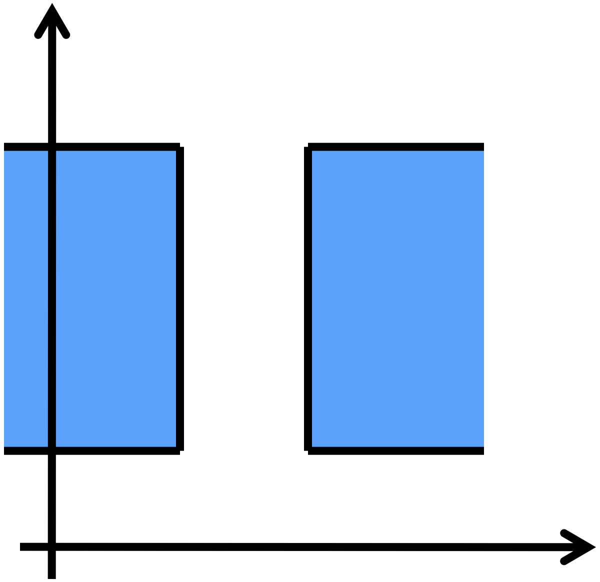
Boxes: Representation





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