Building Program Verifiers from Compilers and Theorem Provers

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The Lab

Download SeaHorn v.0.1.0-rc2

- http://github.com/seahorn/seahorn/releases

(Optionally) If you need a virtual machine, see instructions at:

- http://arieg.bitbucket.org/ssft15.html

Clone http://github.com/seahorn/seahorn-tutoral for examples

PLEASE DO THIS BEFORE THE LAB ☺

For THIS lecture, additional material at

- http://arieg.bitbucket.org/pdf/gurfinkel_ssft15.pdf

Constrained Horn Clauses (CHC)

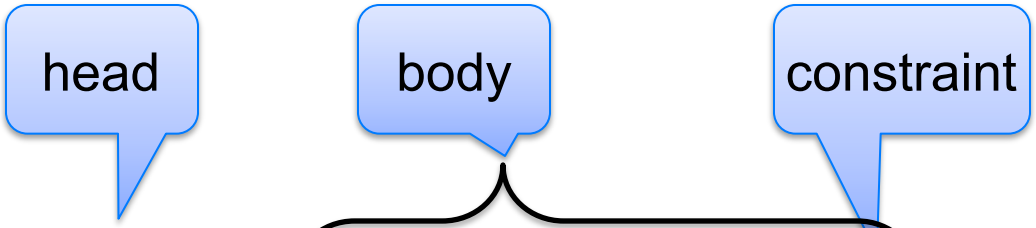
A Constrained Horn Clause (CHC) is a FOL formula of the form

$$\forall V . (\phi \wedge p_1[X_1] \wedge \ldots \wedge p_n[X_n] \rightarrow h[X]),$$

where

- A is a background theory (e.g., Linear Arithmetic, Arrays, Bit-Vectors, or combinations of the above)
- ϕ is a constrained in the background theory A
- $p_1, \ldots, p_n$, h are n-ary predicates
- $p_i[X]$ is an application of a predicate to first-order terms

CHC Terminology



Rule $h[X] \leftarrow p_1[X_1], \ldots, p_n[X_n], \phi.$

Query $\text{false} \leftarrow p_1[X_1], \ldots, p_n[X_n], \phi.$

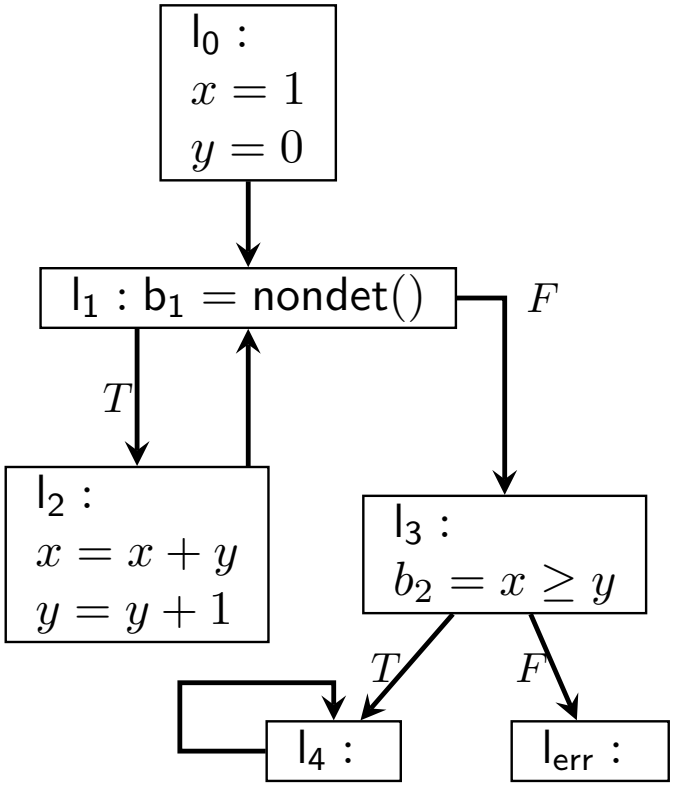
Fact $h[X] \leftarrow \phi.$

Linear CHC $h[X] \leftarrow p[X_1], \phi.$

Non-Linear CHC $h[X] \leftarrow p_1[X_1], \ldots, p_n[X_n], \phi.$ for $n > 1$

Example Horn Encoding

```
int x = 1;
int y = 0;
while (*) {
    x = x + y;
    y = y + 1;
}
assert(x ≥ y);
```



$\langle 1 \rangle$ p_0.

$\langle 2 \rangle$ $\mathsf{p}_1(x, y) \leftarrow$
$\mathsf{p}_0, x = 1, y = 0$.

$\langle 3 \rangle$ $\mathsf{p}_2(x, y) \leftarrow \mathsf{p}_1(x, y)$.

$\langle 4 \rangle$ $\mathsf{p}_3(x, y) \leftarrow \mathsf{p}_1(x, y)$.

$\langle 5 \rangle$ $\mathsf{p}_1(x', y') \leftarrow$
$\mathsf{p}_2(x, y)$,
$x' = x + y$,
$y' = y + 1$.

$\langle 6 \rangle$ $\mathsf{p}_4 \leftarrow (x \geq y), \mathsf{p}_3(x, y)$.

$\langle 7 \rangle$ $\mathsf{p}_{\mathsf{err}} \leftarrow (x < y), \mathsf{p}_3(x, y)$.

$\langle 8 \rangle$ $\mathsf{p}_4 \leftarrow \mathsf{p}_4$.

$\langle 9 \rangle$ $\bot \leftarrow \mathsf{p}_{\mathsf{err}}$.

CHC Satisfiability

A **model** of a set of clauses Π is an interpretation of each predicate p_i that makes all clauses in Π valid

A set of clauses is **satisfiable** if it has a model, and is unsatisfiable otherwise

A model is **A-definable**, it each p_i is definable by a formula ψ_i in A

In the context of program verification

- a program satisfies a property iff corresponding CHCs are satisfiable
- verification certificates correspond to models
- counterexamples correspond to derivations of false

SOLVING CHC WITH SMT

IC3, PDR, and Friends (1)

IC3: A SAT-based Hardware Model Checker

- Incremental Construction of Inductive Clauses for Indubitable Correctness
- A. Bradley: SAT-Based Model Checking without Unrolling. VMCAI 2011

PDR: Explained and extended the implementation

- Property Directed Reachability
- N. Eén, A. Mishchenko, R. K. Brayton: Efficient implementation of property directed reachability. FMCAD 2011

PDR with Predicate Abstraction (easy extension of IC3/PDR to SMT)

- A. Cimatti, A. Griggio, S. Mover, St. Tonetta: IC3 Modulo Theories via Implicit Predicate Abstraction. TACAS 2014
- J. Birgmeier, A. Bradley, G. Weissenbacher: Counterexample to Induction-Guided Abstraction-Refinement (CTIGAR). CAV 2014

IC3, PDR, and Friends (2)

GPDR: Non-Linear CHC with Arithmetic constraints

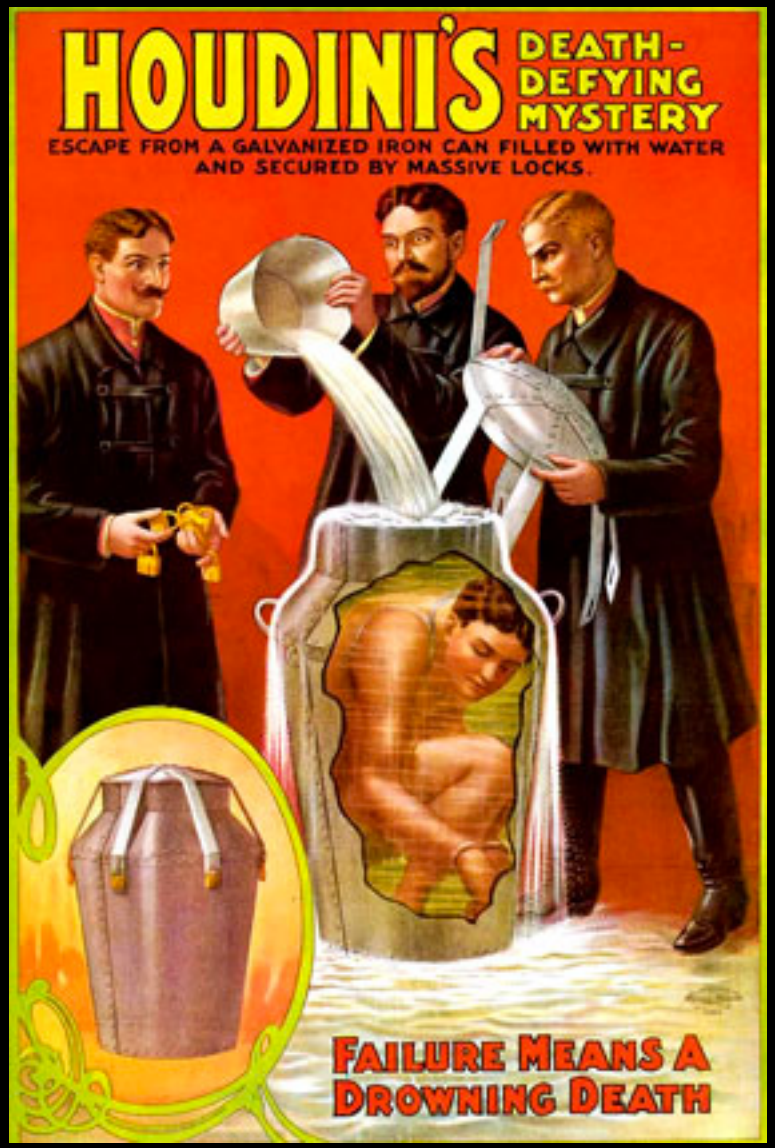
- Generalized Property Directed Reachability
- K. Hoder and N. Bjørner: Generalized Property Directed Reachability. SAT 2012

SPACER: Non-Linear CHC with Arithmetic

- fixes an incompleteness issue in GPDR and extends it with under-approximate summaries
- A. Komuravelli, A. Gurfinkel, S. Chaki: SMT-Based Model Checking for Recursive Programs. CAV 2014

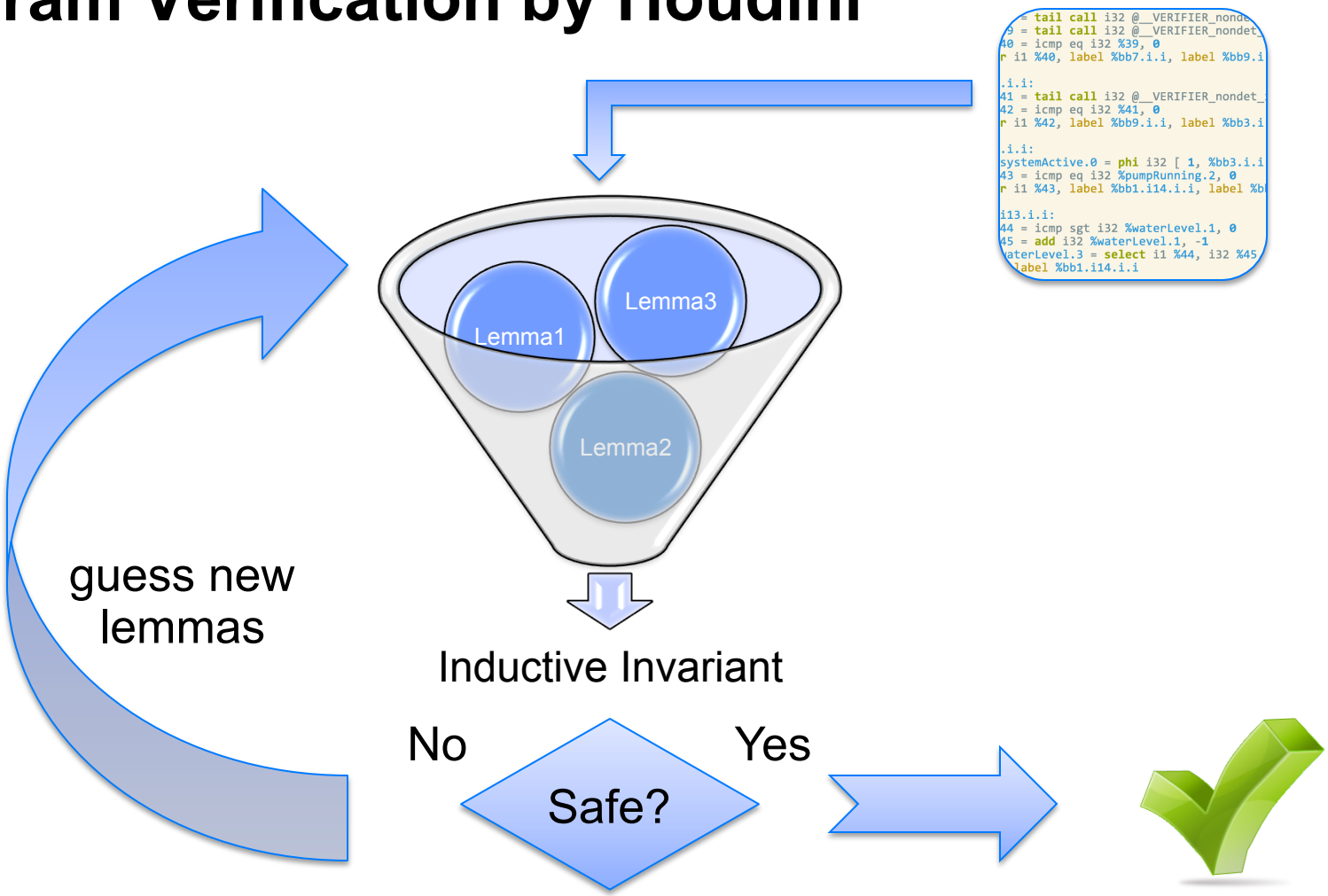
PolyPDR: Convex models for Linear CHC

- simulating Numeric Abstract Interpretation with PDR
- N. Bjørner and A. Gurfinkel: Property Directed Polyhedral Abstraction. VMCAI 2015

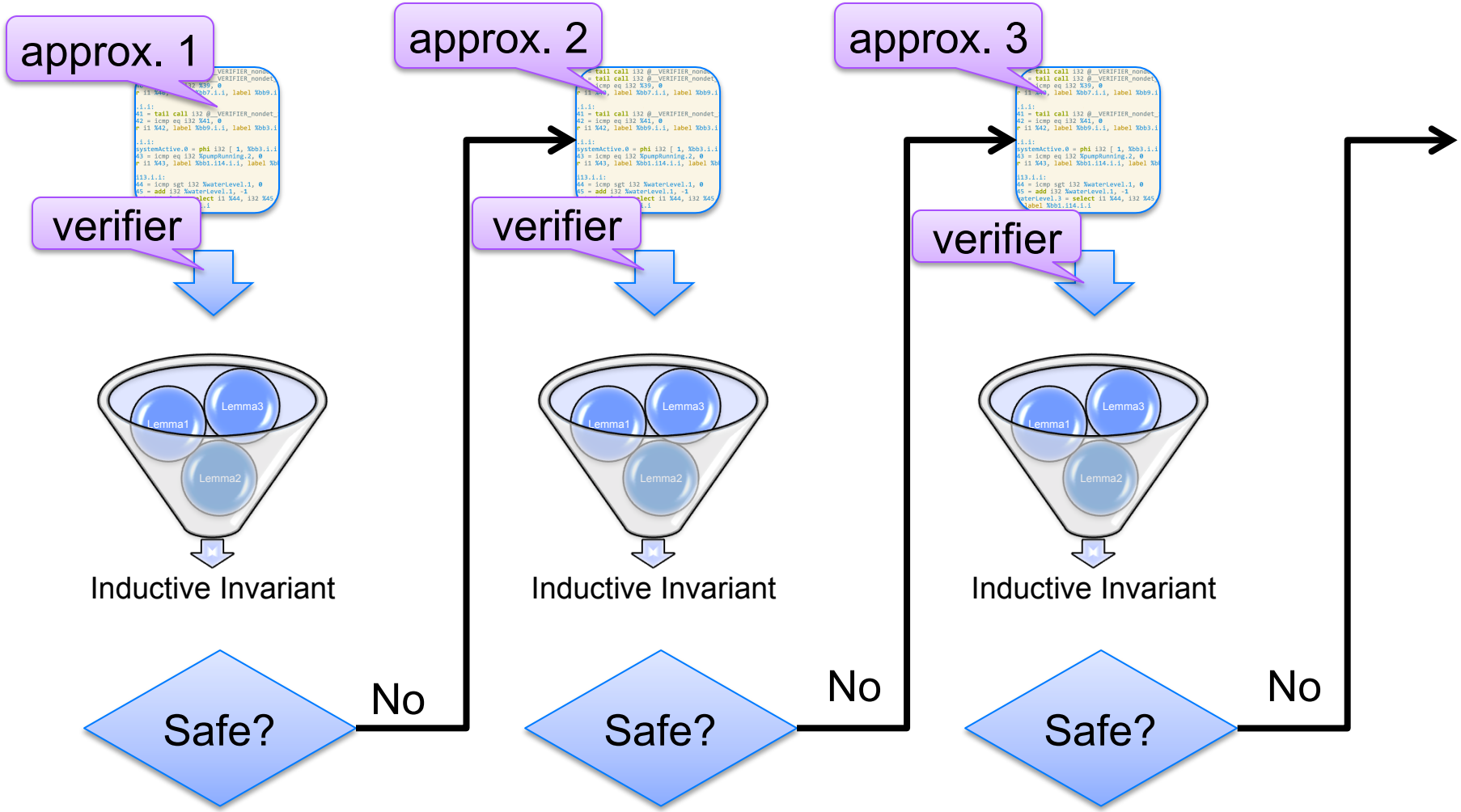


Cormac Flanagan, K. Rustan M. Leino: Houdini, an Annotation Assistant for ESC/Java. FME 2001: 500-517

Program Verification by Houdini



Guessing Lemmas by Evolving Approximations



Linear CHC Satisfiability

Satisfiability of a set of linear CHCs is reducible to satisfiability of THREE clauses of the form

$$Init(X) \rightarrow P(X)$$
$$P(X) \rightarrow Bad(X)$$
$$P(X) \wedge Tr(X, X') \rightarrow P(X')$$

where, X' = {x' | x $\in$ X}, P a fresh predicate, and *Init*, *Bad*, and *Tr* are constraints

Proof:

add extra arguments to distinguish between predicates

$$\frac{Q(y) \wedge \phi \rightarrow W(y, z)}{P(id='Q', y) \wedge \phi \rightarrow P(id='W', y, z)}$$

Programs, Cexs, Invariants

A program *P = (V, Init, $\mathcal{T}r$, Bad)*

- Notation: $\mathcal{F}(X) = \exists\ \boldsymbol{u}\ .\ (X \wedge \mathcal{T}r) \vee \text{Init}$

P is UNSAFE if and only if there exists a number *N* s.t.

$$Init(X_0) \wedge \left(\bigwedge_{i=0}^{N-1} Tr(X_i, X_{i+1}) \right) \wedge Bad(X_N) \not\Rightarrow \bot$$

P is SAFE if and only if there exists a *safe inductive invariant Inv s.t.*

$$\left.\begin{array}{r} Init \Rightarrow Inv \\ Inv(X) \wedge Tr(X, X') \Rightarrow Inv(X') \end{array}\right\} \text{Inductive}$$

$$Inv \Rightarrow \neg Bad \qquad \text{Safe}$$

IC3/PDR Algorithm Overview

Input: Safety problem $\langle Init(X), Tr(X, X'), Bad(X) \rangle$

$F_0 \leftarrow Init$; $N \leftarrow 0$ **repeat**

  $\mathbf{G} \leftarrow \textsc{PdrMkSafe}([F_0, \ldots, F_N], Bad)$

  if $\mathbf{G} = [\,]$ **then return** *Reachable*;

  $\forall 0 \leq i \leq N \cdot F_i \leftarrow \mathbf{G}[i]$

  $F_0, \ldots, F_N \leftarrow \textsc{PdrPush}([F_0, \ldots, F_N])$

  if $\exists 0 \leq i < N \cdot F_i = F_{i+1}$ **then return** *Unreachable*;

  $N \leftarrow N + 1$; $F_N \leftarrow \emptyset$

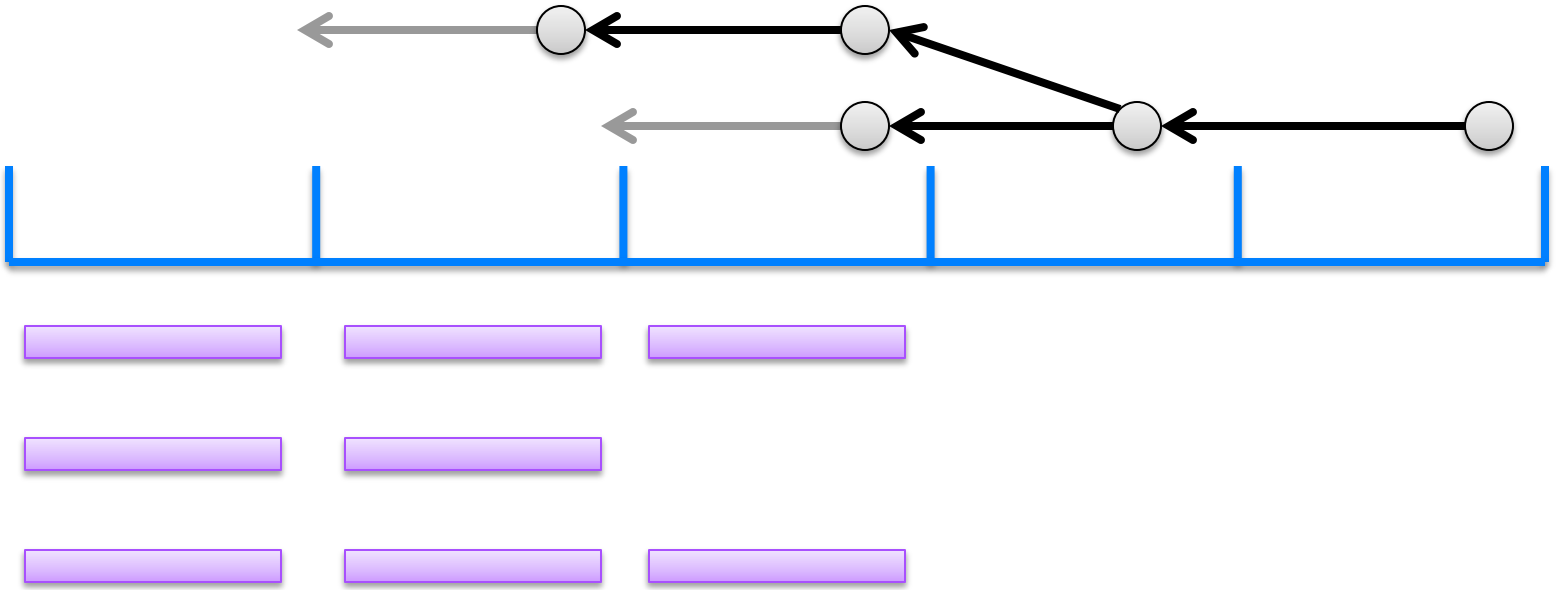
until ∞;

bounded safety

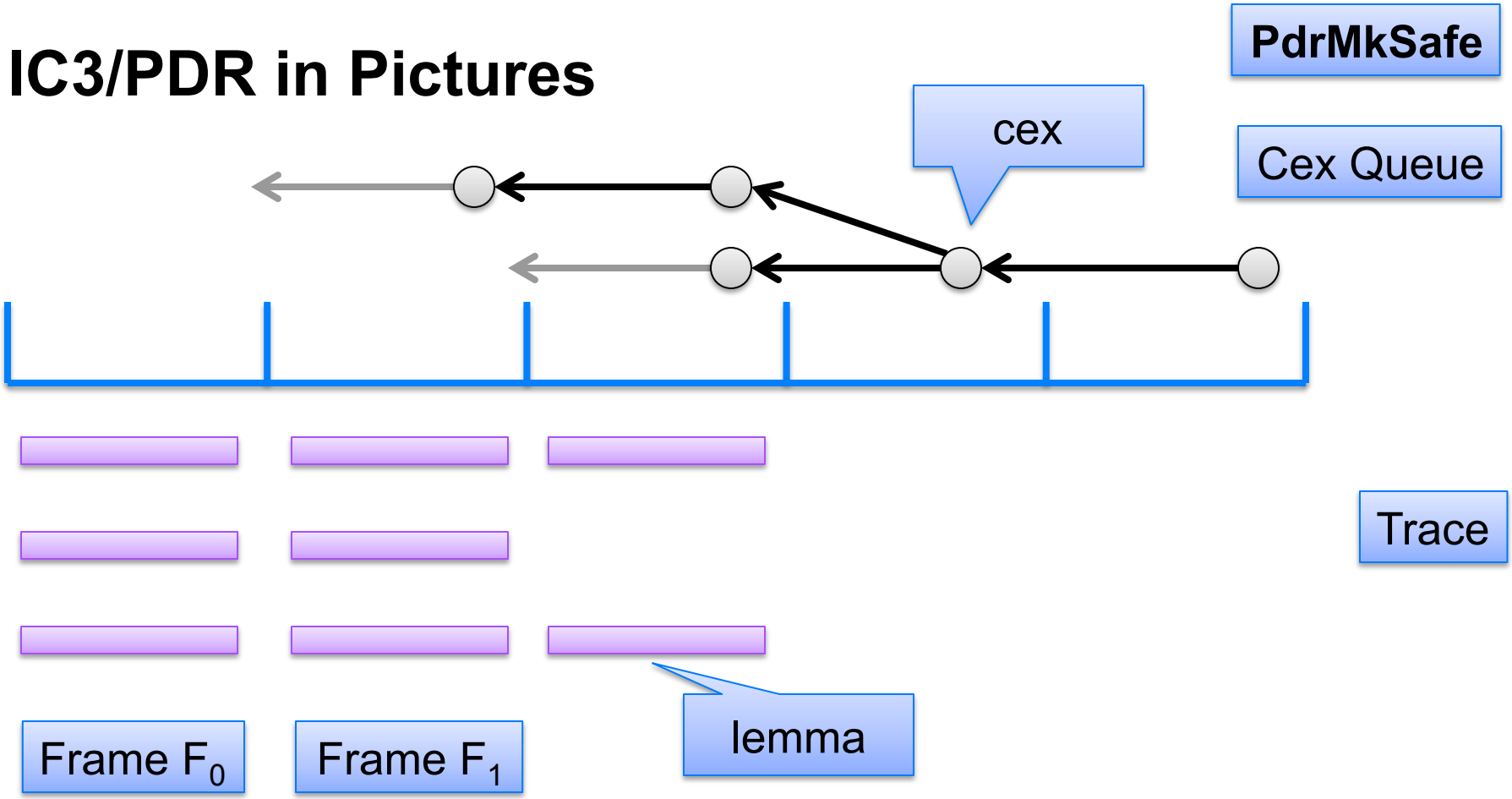
strengthen result

IC3/PDR in Pictures

PdrMkSafe

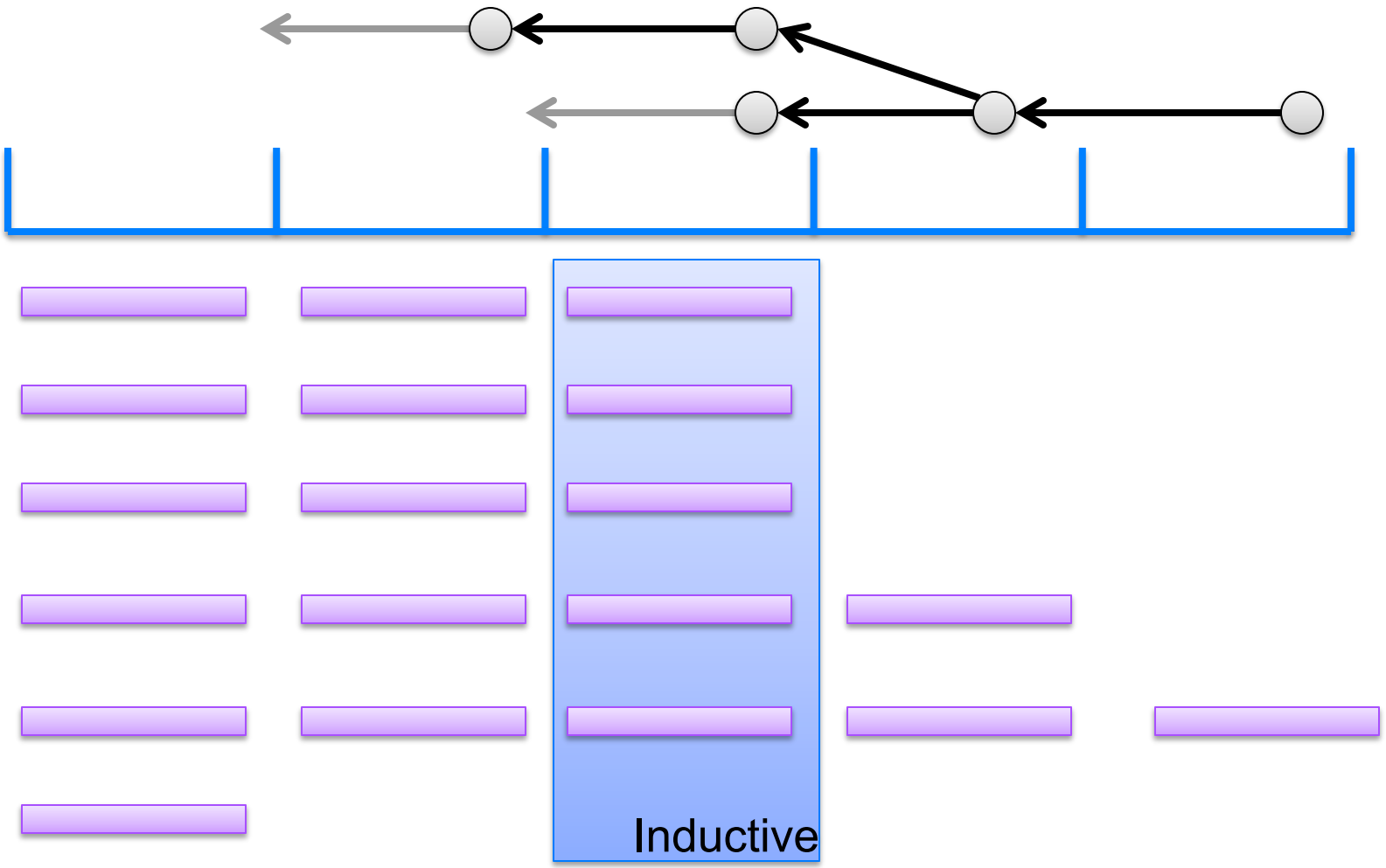


IC3/PDR in Pictures



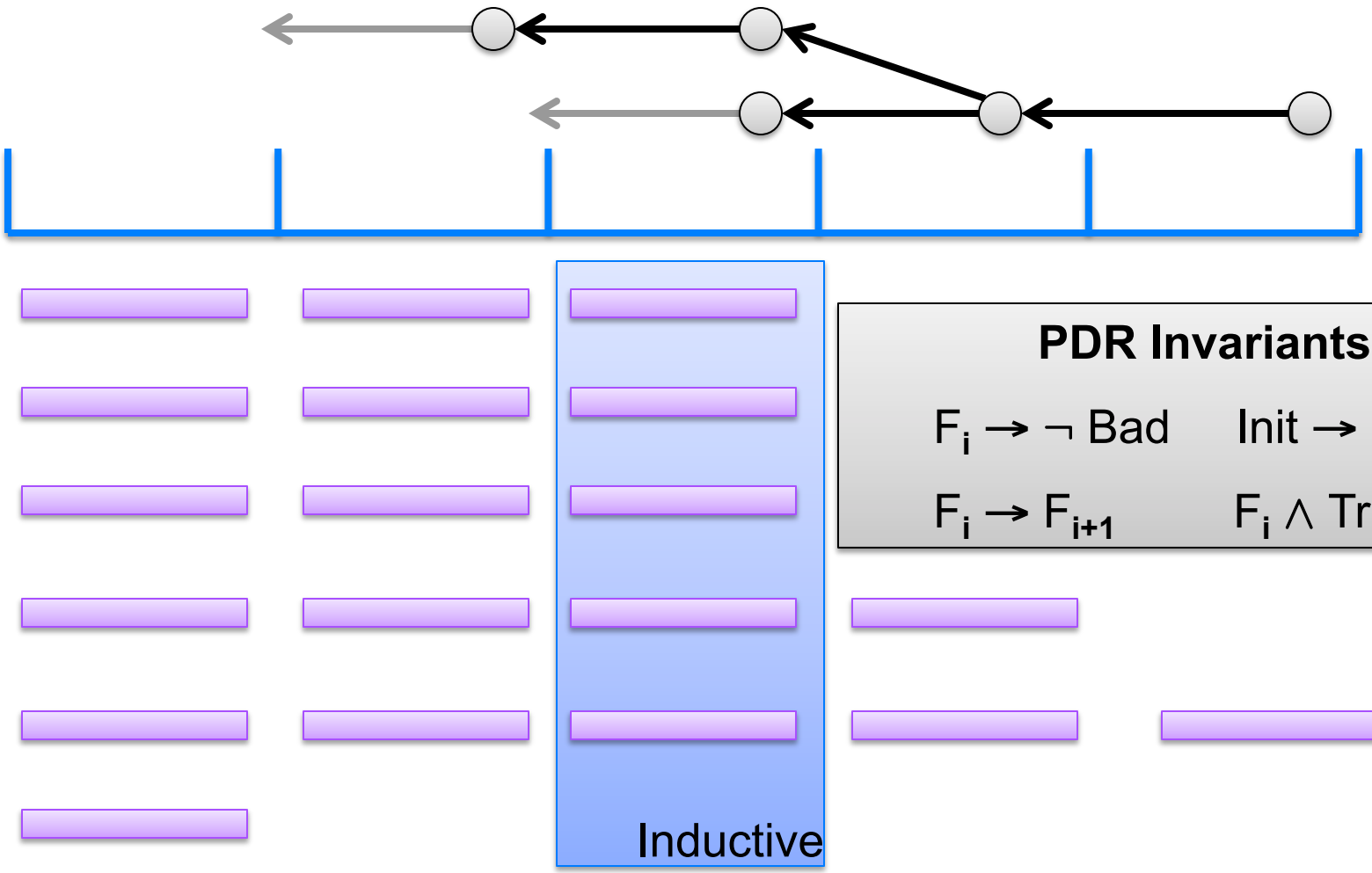
IC3/PDR in Pictures

PdrPush



IC3/PDR in Pictures

PdrPush



PDR Invariants

$F_i \rightarrow \neg$ Bad Init $\rightarrow F_i$

$F_i \rightarrow F_{i+1}$ $F_i \wedge Tr \rightarrow F_{i+1}$

IC3/PDR

Input: A safety problem $\langle Init(X), Tr(X, X'), Bad(X)\rangle$.
Output: *Unreachable* or *Reachable*
Data: A cex queue Q, where $c \in Q$ is a pair $\langle m, i\rangle$, m is a cube over state variables, and $i \in \mathbb{N}$. A level N. A trace $F_0, F_1, \ldots$
Initially: $Q = \emptyset$, $N = 0$, $F_0 = Init$, $\forall i > 0 \cdot F_i = \emptyset$.
repeat

Unreachable If there is an $i < N$ s.t. $F_i \subseteq F_{i+1}$ **return** *Unreachable*.

Reachable If there is an m s.t. $\langle m, 0\rangle \in Q$ **return** *Reachable*.

Unfold If $F_N \rightarrow \neg Bad$, then set $N \leftarrow N + 1$.

Candidate If for some m, $m \rightarrow F_N \wedge Bad$, then add $\langle m, N\rangle$ to Q.

Decide If $\langle m, i + 1\rangle \in Q$ and there are m_0 and m_1 s.t. $m_1 \rightarrow m$, $m_0 \wedge m_1'$ is satisfiable, and $m_0 \wedge m_1' \rightarrow F_i \wedge Tr \wedge m'$, then add $\langle m_0, i\rangle$ to Q.

Conflict For $0 \leq i < N$: given a candidate model $\langle m, i + 1\rangle \in Q$ and clause φ, such that $\varphi \rightarrow \neg m$, if $Init \rightarrow \varphi$, and $\varphi \wedge F_i \wedge Tr \rightarrow \varphi'$, then add φ to F_j, for $j \leq i + 1$.

Leaf If $\langle m, i\rangle \in Q$, $0 < i < N$ and $F_{i-1} \wedge Tr \wedge m'$ is unsatisfiable, then add $\langle m, i + 1\rangle$ to Q.

Induction For $0 \leq i < N$ and a clause $(\varphi \vee \psi) \in F_i$, if $\varphi \notin F_{i+1}$, $Init \rightarrow \varphi$ and $\varphi \wedge F_i \wedge Tr \rightarrow \varphi'$, then add φ to F_j, for each $j \leq i + 1$.

until ∞;

IC3 Data-Structures

A trace $F = F_0, \ldots, F_N$ is a sequence of frames.

- A frame F_i is a set of clauses. Elements of F_i are called lemmas.
- Invariants:
  - Bounded Safety: $\forall i < N . F_i \rightarrow \neg Bad$
  - Monotonicity: $\forall i < N . F_{i+1} \subseteq F_i$
  - Inductiveness: $\forall i < N . F_i \wedge Tr \rightarrow F'_{i+1}$

A priority queue Q of counterexamples to induction (CTI)

- $(m, i) \in Q$ is a pair, where m is a cube and i a level
- if $(m, i) \in Q$ then there exists a path of length (N-i) from a state in m to a state in Bad
- Q is ordered by level
  - $(m, i) < (k, j)$ iff $i < j$

Termination and Progress

Unreachable If there is an $i < N$ s.t. $F_i \subseteq F_{i+1}$
return *Unreachable*.

Reachable If there is an m s.t. $\langle m, 0 \rangle \in Q$
return *Reachable*.

Unfold If $F_N \rightarrow \neg Bad$, then set $N \leftarrow N + 1$.

Candidate If for some m, $m \rightarrow F_N \wedge Bad$,
then add $\langle m, N \rangle$ to Q.

Inductive Generalization

Conflict For $0 \leq i < N$: given a candidate model $\langle m, i+1 \rangle \in Q$ and clause φ, such that $\varphi \rightarrow \neg m$, if $Init \rightarrow \varphi$, and $\varphi \wedge F_i \wedge Tr \rightarrow \varphi'$, then add φ to F_j, for $j \leq i+1$.

A clause φ is inductive relative to F iff

- Init → φ (Initialization) and φ ∧ F ∧ Tr → φ' (Inductiveness)

Implemented by first letting φ = ¬m and generalizing φ by iteratively dropping literals while checking the inductiveness condition

Theorem: Let $F_0, F_1, \ldots, F_N$ be a valid IC3 trace. If φ is inductive relative to F_i, $0 \leq i < N$, then, for all $j \leq i$, φ is inductive relative to F_j.

- Follows from the monotonicity of the trace
  - if $j < i$ then $F_j \rightarrow F_i$
  - if $F_j \rightarrow F_i$ then $(\varphi \wedge F_i \wedge Tr \rightarrow \varphi') \rightarrow (\varphi \wedge F_j \wedge Tr \rightarrow \varphi')$

Prime Implicants

A formula φ is an *implicant* of a formula psi iff $\varphi \Rightarrow \psi$

A *propositional implicant* of ψ is a conjunction of literals φ such that φ is an implicant of ψ

- φ is a conjunction of literals
- $\varphi \Rightarrow \psi$
- φ is a partial assignment that makes ψ true

A propositonal implicant φ of ψ is called *prime* if no subset of φ is an implicant of ψ

- φ is a conjunction of literals
- $\varphi \Rightarrow \psi$
- $\forall\, p\, .\, (p \neq \varphi \wedge \varphi \Rightarrow p) \Rightarrow (p \not\Rightarrow \psi)$

Generalizing Predecessors

Decide If $\langle m, i+1 \rangle \in Q$ and there are m_0 and m_1 s.t. $m_1 \rightarrow m$, $m_0 \wedge m_1'$ is satisfiable, and $m_0 \wedge m_1' \rightarrow F_i \wedge Tr \wedge m'$, then add $\langle m_0, i \rangle$ to Q.

Decide rule chooses a (generalized) predecessor m_0 of m that is consistent with the current frame

Simplest implementation is to extract a predecessor m_o from a satisfying assignment of $M \models Fi \wedge Tr \wedge m'$

- m_0 cab be further generalized using ternary simulation by dropping literals and checking that m' remains forced

An alternative is to let m_0 be an implicant (not necessarily prime) of $F_i \wedge \exists X'.(Tr \wedge m')$

- finding a prime implicant is difficult because of the existential quantification
- we settle for an arbitrary implicant. The side conditions ensure it is not trivial

Strengthening a trace

Induction For $0 \leq i < N$ and a clause $(\varphi \vee \psi) \in F_i$, if $\varphi \notin F_{i+1}$, $Init \rightarrow \varphi$ and $\varphi \wedge F_i \wedge Tr \rightarrow \varphi'$, then add φ to F_j, for each $j \leq i + 1$.

Also known as **Push** or **Propagate**

Bounded safety proofs are usually very weak towards the end

- not much is needed to show that error will not happen in one or two steps

This tends to make them non-inductive

- a weakness of interpolation-based model checking, like IMPACT
- in IMPACT, this is addressed by forced covering heuristic

Induction "applies" forced cover one lemma at a time

- whenever all lemmas are pushed F_{i+1} is inductive (and safe)
- (optionally) combine strengthening with generalization

Implementation

- Apply Induction from 0 to N whenever **Conflict** and **Decide** are not applicable

Long Counterexamples

Leaf If $\langle m, i \rangle \in Q$, $0 < i < N$ and $F_{i-1} \wedge Tr \wedge m'$ is unsatisfiable, then add $\langle m, i+1 \rangle$ to Q.

Whenever a counterexample *m* is blocked at level *i*, it is known that

- there is no path of length *i* from *Init* to *m* (because got blocked)
- there is a path of length *(N-i)* from *m* to *Bad*

Can check whether there exists a path of length *(i+1)* from *Init* to *m*

- (**Leaf**) check eagerly by placing the CTI back into the queue at a higher level
- **(No Leaf)** check lazily by waiting until the same (or similar) CTI is discovered after N is increased by **Unfold**

Leaf allows IC3 to discover counterexamples much longer than the current unfolding depth N

- each CTI re-enqueued by **Leaf** adds one to the depth of the longest possible counterexample found
- a real counterexample might chain through multiple such CTI's

Queue Management for Long Counterexamples

A queue element is a triple *(m, i, d)*

- *m* is a CTI, *i* a level, *d* a depth

Decide sets *m* and *i* as before, and sets *d* to 0

Leaf increases *i* and *d* by one

- *i* determines how far the CTI can be pushed back
- *d* counts number of times the CTI was pushed forward

Queue is ordered first by level, then by depth

- $(m, i, d) < (k, j, e) \Leftrightarrow i < j \vee (i=j \wedge d < e)$

Overall exploration mimics iterative deepening with non-uniform exploration depth

- go deeper each time before backtracking

PDR FOR ARITHMETIC

Arithmetic PDR

Input: A safety problem $\langle Init(X), Tr(X, X'), Bad(X)\rangle$.
Output: *Unreachable* or *Reachable*
Data: A cex queue Q, where $c \in Q$ is a pair $\langle m, i\rangle$, m is a cube over state variables, and $i \in \mathbb{N}$. A level N. A trace $F_0, F_1, \ldots$
Initially: $Q = \emptyset$, $N = 0$, $F_0 = Init$, $\forall i > 0 \cdot F_i = \emptyset$.
repeat
  Unreachable If there is an $i < N$ s.t. $F_i \subseteq F_{i+1}$ **return** *Unreachable*.

Notation: $\mathcal{F}(A) = A(X) \wedge Tr(X, X') \vee Init(X')$.

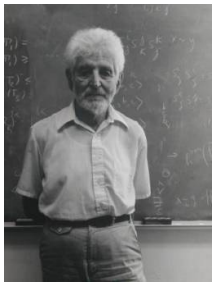
Decide If $\langle P, i+1\rangle \in Q$ and there is a model $m(X, X')$ s.t. $m \models \mathcal{F}(F_i) \wedge P'$, add $\langle P_\downarrow, i\rangle$ to Q, where $P_\downarrow = \mathrm{MBP}(X', m, \mathcal{F}(F_i) \wedge P')$.

Conflict For $0 \leq i < N$, given a counterexample $\langle P, i+1\rangle \in Q$ s.t. $\mathcal{F}(F_i) \wedge P'$ is unsatisfiable, add $P^\uparrow = \textsc{Itp}(\mathcal{F}(F_i)(X^o, X), P)$ to F_j for $j \leq i+1$.

  Induction For $0 \leq i < N$ and a clause $(\varphi \vee \psi) \in F_i$, if $\varphi \notin F_{i+1}$, $Init \rightarrow \varphi$ and $\varphi \wedge F_i \wedge Tr \rightarrow \varphi'$, then add φ to F_j, for each $j \leq i+1$.

until ∞;

Craig Interpolation Theorem



Theorem (Craig 1957)

Let A and B be two First Order (FO) formulae such that $A \Rightarrow \neg B$, then there exists a FO formula I, denoted ITP(A, B), such that

$$A \Rightarrow I \qquad I \Rightarrow \neg B$$

$$atoms(I) \in atoms(A) \cap atoms(B)$$

A Craig interpolant ITP(A, B) can be effectively constructed from a resolution proof of unsatisfiability of $A \wedge B$

In Model Cheching, Craig Interpolation Theorem is used to safely over-approximate the set of (finitely) reachable states

Alternative Definition of an Interpolant

Let F = A(x, z) $\wedge$ B(z, y) be UNSAT, where x and y are distinct

- Note that for any assignment v to z either
  - A(x, v) is UNSAT, or
  - B(v, y) is UNSAT

An interpolant is a circuit I(z) such that for every assignment v to z

- I(v) = A only if A(x, v) is UNSAT
- I(v) = B only if B(v, y) is UNSAT

A proof system S has a *feasible interpolation* if for every refutation π of F in S, F has an interpolant polynomial in the size of π

- propositional resolution has feasible interpolation
- extended resolution does not have feasible interpolation

Farkas Lemma

Let $M = t_1 \geq b_1 \wedge \ldots \wedge t_n \geq b_n$, where t_i are linear terms and b_i are constants M is *unsatisfiable* iff $0 \geq 1$ is derivable from M by resolution

M is *unsatisfiable* iff $M \vdash 0 \geq 1$

- e.g., $x + y > 10, -x > 5, -y > 3 \vdash (x+y-x-y) > (10 + 5 + 3) \vdash 0 > 18$

M is unsatisfiable iff there exist *Farkas* coefficients $g_1, \ldots, g_n$ such that

- $g_i \geq 0$
- $g_1 \times t_1 + \ldots + g_n \times t_n = 0$
- $g_1 \times b_1 + \ldots + g_n \times b_n \geq 1$

Interpolation for Linear Real Arithmetic

Let $M = A \wedge B$ be UNSAT, where

- $A = t_1 \geq b_1 \wedge \ldots \wedge t_i \geq b_i$, and
- $B = t_{i+1} \geq b_i \wedge \ldots \wedge t_n \geq b_n$

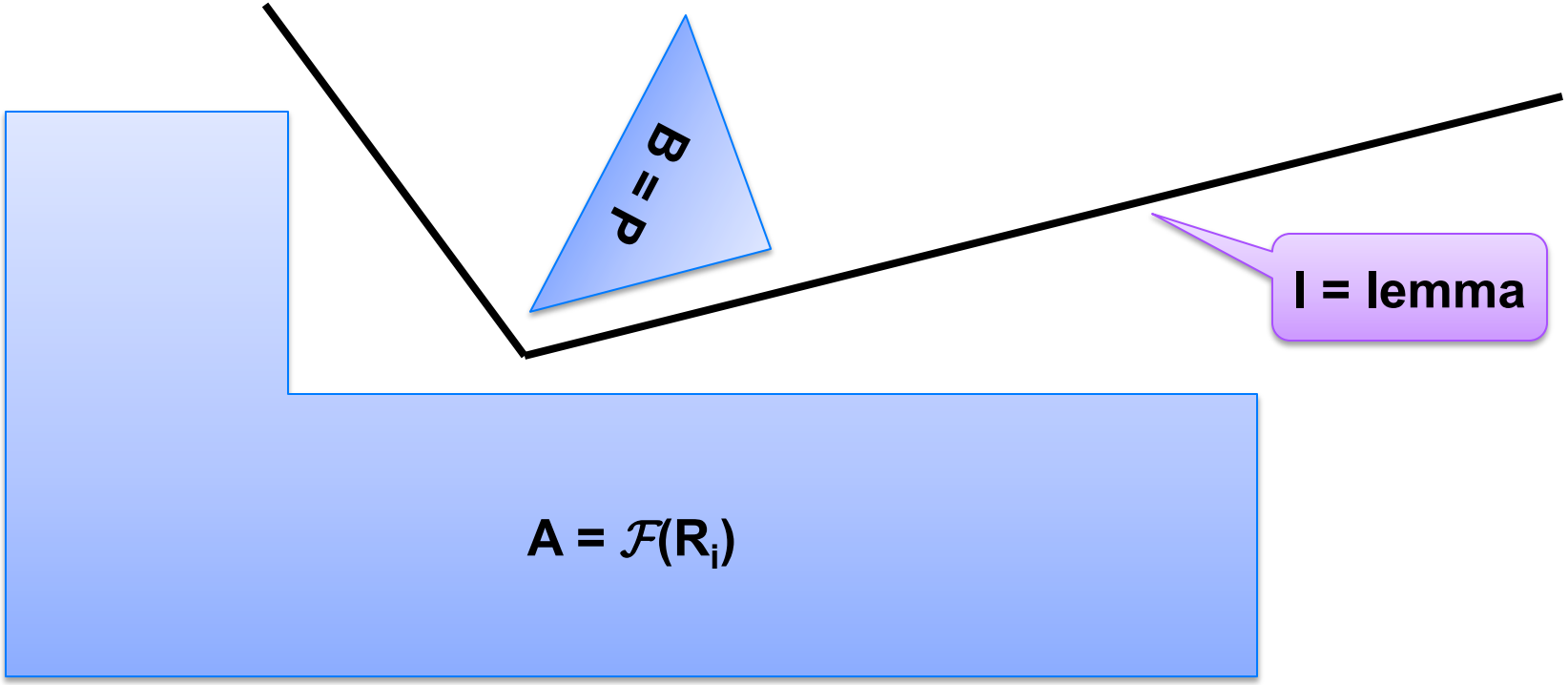
Let $g_1, \ldots, g_n$ be the Farkas coefficients witnessing UNSAT

Then

- $g_1 \times (t_1 \geq b_1) + \ldots + g_i \times (t_i \geq b_i)$ is an interpolant between A and B
- $g_{i+1} \times (t_{i+1} \geq b_i) + \ldots + g_n \times (t_n \geq b_n)$ is an interpolant between B and A

- $g_1 \times t_1 + \ldots + g_i \times t_i = -(g_{i+1} \times t_{i+1} + \ldots + g_n \times t_n)$
- $\neg(g_{i+1} \times (t_{i+1} \geq b_i) + \ldots + g_n \times (t_n \geq b_n))$ is an interpolant between A and B

Craig Interpolation for Linear Arithmetic



Useful properties of existing interpolation algorithms [CGS10] [HB12]

- $I \in ITP(A, B)$ then $\neg I \in ITP(B, A)$
- if A is syntactically convex (a monomial), then I is convex
- if B is syntactically convex, then I is co-convex (a clause)
- if A and B are syntactically convex, then I is a half-space

Arithmetic Conflict

Notation: $\mathcal{F}(A) = (A(X) \wedge Tr) \vee Init(X')$.

Conflict For $0 \leq i < N$, given a counterexample $\langle P, i+1 \rangle \in Q$ s.t. $\mathcal{F}(F_i) \wedge P'$ is unsatisfiable, add $P^{\uparrow} = \textsc{Itp}(\mathcal{F}(F_i), P')$ to F_j for $j \leq i+1$.

Counterexample is blocked using Craig Interpolation

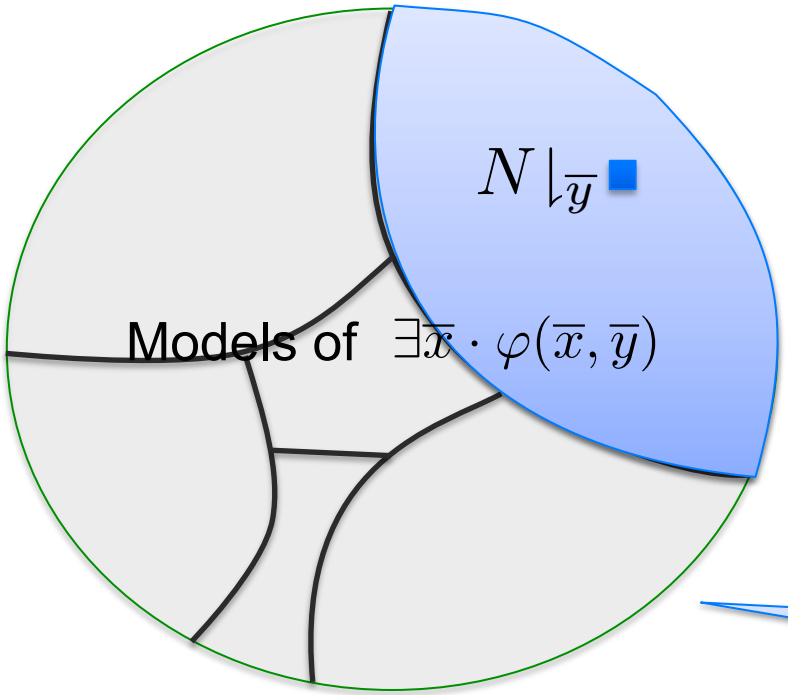
- summarizes the reason why the counterexample cannot be extended

Generalization is not inductive

- weaker than IC3/PDR
- inductive generalization for arithmetic is still an open problem

Model Based Projection

Expensive to find a quantifier-free $\psi(\overline{y}) \equiv \exists \overline{x} \cdot \varphi(\overline{x}, \overline{y})$



1. find $N \models \varphi(\overline{x}, \overline{y})$

(e.g. specific pre-post pair that needs to be generalized)

2. choose disjunct "covering" *N*

using virtual substitution

Lazy Quantifier Elimination!

Model Based Projection

Definition: Let φ be a formula, U a set of variables, and M a model of φ. Then ψ = MBP (U, M, φ) is a Model Based Project of U, M and φ iff

1. ψ is a monomial (optional)
2. Vars(ψ) $\subseteq$ Vars(φ) $\setminus$ U
3. M $\models \psi$
4. $\psi \Rightarrow \exists$ U . φ

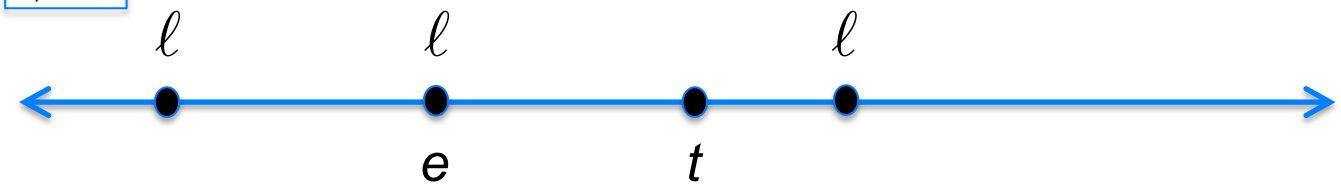
For a fixed set of variables U and a formula φ, MBP is a function from models to formulas

MBP is *finite* if its range (as a function defined above) is finite

MBP for Linear Rational Arithmetic

$$\exists \ell \cdot (\ell = e \wedge \phi_1) \vee (t < \ell \wedge \ell < u) \vee (\ell < u \wedge \phi_2)$$

$$\equiv \quad (\phi_1 \vee (t < e \wedge e < u) \vee (e < u \wedge \phi_2))$$
$$\vee \; (t < u \vee (t < u \wedge \phi_2))$$
$$\vee \; \phi_2$$



pick a disjunct that covers a given model

[1] Loos and Weispfenning, *Applying Linear Quantifier Elimination*, 1993
[2] Tobias Nipkow, *Linear Quantifier Elimination*, 2008
[3] Bjorner, *Linear Quantifier Elimination as an Abstract Decision Procedure*, 2010

Arithmetic Decide

Notation: $\mathcal{F}(A) = (A(X) \wedge Tr(X, X') \vee Init(X')$.

Decide If $\langle P, i+1 \rangle \in Q$ and there is a model $m(X, X')$ s.t. $m \models \mathcal{F}(F_i) \wedge P'$,
add $\langle P_{\downarrow}, i \rangle$ to Q, where $P_{\downarrow} = \mathrm{MBP}(X', m, \mathcal{F}(F_i) \wedge P')$.

Compute a predecessor using an under-approximation of quantifier elimination – called Model Based Projection

To ensure progress, Decide must be finite

- finitely many possible predecessors when all other arguments are fixed

Alternatives

- Completeness can follow from the **Conflict** rule only
  - for Linear Arithmetic this means using Fourier-Motzkin implicants
- Completeness can follow from an interaction of **Decide** and **Conflict**

PDR FOR NON-LINEAR CHC

Non-Linear CHC Satisfiability

Satisfiability of a set of arbitrary (i.e., linear or non-linear) CHCs is reducible to satisfiability of THREE clauses of the form

$$Init(X) \rightarrow P(X)$$

$$P(X) \rightarrow Bad(X)$$

$$P(X) \wedge P(X^o) \wedge Tr(X, X^o, X') \rightarrow P(X')$$

where, X' = {x' | x ∈ X}, X^o = {x^o | x ∈ X}, P a fresh predicate, and Init, Bad, and Tr are constraints

Proof:

- factor rules with more than 2 predicates in the body

  replace $P_1(x) \wedge P_2(y) \wedge P_3(z) \wedge \phi(x,y,z) \rightarrow H(x, y, z)$

  by $P_1(x) \wedge W(y,z) \wedge phi(x,y,z) \rightarrow H(x,y,z)$. $P_2(y) \wedge P_3(z) \rightarrow W(y, z)$.
- add extra arguments to distinguish between predicates

  $P(id='P_2', y) \wedge P(id='P_3', z) \rightarrow P(id='W', y, z)$.

Non-linear CHC by reduction to linear CHC

Can non-linear CHC satisfiability be reduced to (multiple) linear CHC satisfiability problems?

$$Init(X) \rightarrow P(X)$$
$$P(X) \rightarrow Bad(X)$$
$$P(X) \wedge P(X^o) \wedge Tr(X, X^o, X') \rightarrow P(X')$$

Generalized GPDR

Input: A safety problem $\langle Init(X), Tr(X, X^o, X'), Bad(X)\rangle$.
Output: *Unreachable* or *Reachable*
Data: A cex queue Q, where a cex $\langle c_0, \ldots, c_k\rangle \in Q$ is a tuple, each $c_j = \langle m, i\rangle$, m is a cube over state variables, and $i \in \mathbb{N}$. A level N. A trace $F_0, F_1, \ldots$
Notation: $\mathcal{F}(A, B) = Init(X') \vee (A(X) \wedge B(X^o) \wedge Tr)$, and $\mathcal{F}(A) = \mathcal{F}(A, A)$
Initially: $Q = \emptyset$, $N = 0$, $F_0 = Init$, $\forall i > 0 \cdot F_i = \emptyset$
Require: $Init \rightarrow \neg Bad$
repeat

Unreachable If there is an $i < N$ s.t. $F_i \subseteq F_{i+1}$ **return** *Unreachable*.

Reachable if exists $t \in Q$ s.t. for all $\langle c, i\rangle \in t$, $i = 0$, **return** *Reachable*.

Unfold If $F_N \rightarrow \neg Bad$, then set $N \leftarrow N + 1$ and $Q \leftarrow \emptyset$.

Candidate If for some m, $m \rightarrow F_N \wedge Bad$, then add $\langle\langle m, N\rangle\rangle$ to Q.

Decide If there is a $t \in Q$, with $c = \langle m, i+1\rangle \in t$, $m_1 \rightarrow m$, $l_0 \wedge m_0^o \wedge m_1'$ is satisfiable, and $l_0 \wedge m_0^o \wedge m_1' \rightarrow F_i \wedge F_i^o \wedge Tr \wedge m'$ then add $\hat{t}$ to Q, where $\hat{t} = t$ with c replaced by two tuples $\langle l_0, i\rangle$, and $\langle m_0, i\rangle$.

Conflict If there is a $t \in Q$ with $c = \langle m, i+1\rangle \in t$, s.t. $\mathcal{F}(F_i) \wedge m'$ is unsatisfiable. Then, add $\varphi = \textsc{Itp}(\mathcal{F}(F_i), m')$ to F_j, for all $0 \leq j \leq i+1$.

Leaf If there is $t \in Q$ with $c = \langle m, i\rangle \in t$, $0 < i < N$ and $\mathcal{F}(F_{i-1}) \wedge m'$ is unsatisfiable, then add $\hat{t}$ to Q, where $\hat{t}$ is t with c replaced by $\langle m, i+1\rangle$.

Induction For $0 \leq i < N$ and a clause $(\varphi \vee \psi) \in F_i$, if $\varphi \notin F_{i+1}$, $\mathcal{F}(\phi \wedge F_i) \rightarrow \phi'$, then add φ to F_j, for all $j \leq i+1$.

until ∞;

counterexample is a tree

two predecessors

theory-aware **Conflict**

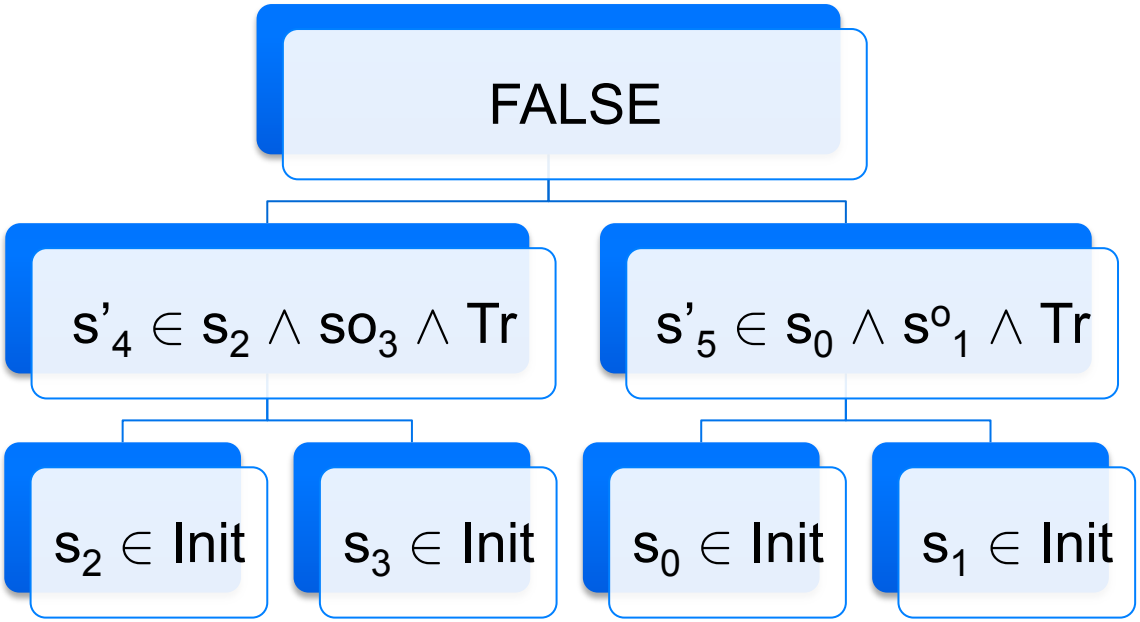
Counterexamples to non-linear CHC

A set S of CHC is unsatisfiable iff S can derive FALSE

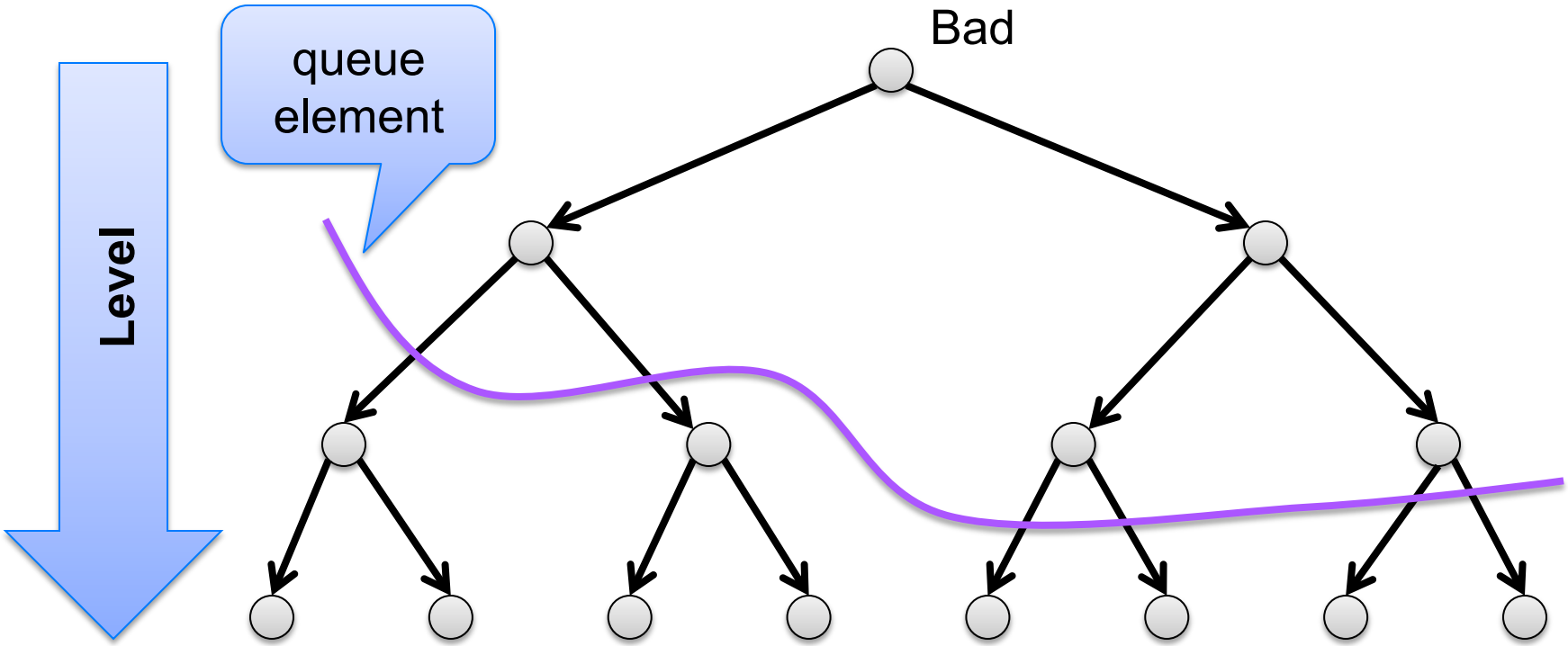
- we call such a derivation a counterexample

For linear CHC, the counterexample is a path

For non-linear CHC, the counterexample is a tree



GPDR Search Space



At each step, one CTI in the frontier is chosen and its two children are expanded

GPDR: Deciding predecessors

Decide If there is a $t \in Q$, with $c = \langle m, i+1 \rangle \in t$, $m_1 \rightarrow m$, $l_0 \wedge m_0^o \wedge m_1'$ is satisfiable, and $l_0 \wedge m_0^o \wedge m_1' \rightarrow F_i \wedge F_i^o \wedge Tr \wedge m'$ then add $\hat{t}$ to Q, where $\hat{t} = t$ with c replaced by two tuples $\langle l_0, i \rangle$, and $\langle m_0, i \rangle$.

Compute two predecessors at each application of **GPDR/Decide**

Can explore both predecessors in parallel

- e.g., BFS or DFS exploration order

Number of predecessors is unbounded

- incomplete even for finite problem (i.e., non-recursive CHC)
- Is compatible with MBP approach of APDR?

No caching/summarization of previous decisions

- worst-case exponential for Boolean Push-Down Systems

Spacer

Same queue as in IC3/PDR

Cache Reachable states

Three variants of **Decide**

Same **Conflict** as in APDR/GPDR

Input: A safety problem $\langle Init(X), Tr(X, X^o, X'), Bad(X)\rangle$.
Output: *Unreachable* or *Reachable*
Data: A cex queue Q, where a cex $c \in Q$ is a pair $\langle m, i\rangle$, m is a cube over state variables, and $i \in \mathbb{N}$. A level N. A set of reachable states REACH. A trace $F_0, F_1, \ldots$
Notation: $\mathcal{F}(A, B) = Init(X') \vee (A(X) \wedge B(X^o) \wedge Tr)$, and $\mathcal{F}(A) = \mathcal{F}(A, A)$
Initially: $Q = \emptyset$, $N = 0$, $F_0 = Init$, $\forall i > 0 \cdot F_i = \emptyset$, REACH $= Init$
Require: $Init \rightarrow \neg Bad$
repeat

Unreachable If there is an $i < N$ s.t. $F_i \subseteq F_{i+1}$ **return** *Unreachable*.

Reachable If REACH $\wedge$ *Bad* is satisfiable, **return** *Reachable*.

Unfold If $F_N \rightarrow \neg Bad$, then set $N \leftarrow N + 1$ and $Q \leftarrow \emptyset$.

Candidate If for some m, $m \rightarrow F_N \wedge Bad$, then add $\langle m, N\rangle$ to Q.

Successor If there is $\langle m, i+1\rangle \in Q$ and a model M $M \models \psi$, where $\psi = \mathcal{F}(\vee\text{REACH}) \wedge m'$. Then, add s to REACH, where $s' \in \text{MBP}(\{X, X^o\}, \psi)$.

DecideMust If there is $\langle m, i+1\rangle \in Q$, and a model M $M \models \psi$, where $\psi = \mathcal{F}(F_i, \vee\text{REACH}) \wedge m'$. Then, add s to Q, where $s \in \text{MBP}(\{X^o, X'\}, \psi)$.

DecideMay If there is $\langle m, i+1\rangle \in Q$ and a model M $M \models \psi$, where $\psi = \mathcal{F}(F_i) \wedge m'$. Then, add s to Q, where $s^o \in \text{MBP}(\{X, X'\}, \psi)$.

Conflict If there is an $\langle m, i+1\rangle \in Q$, s.t. $\mathcal{F}(F_i) \wedge m'$ is unsatisfiable. Then, add $\varphi = \text{ITP}(\mathcal{F}(F_i), m')$ to F_j, for all $0 \leq j \leq i+1$.

Leaf If $\langle m, i\rangle \in Q$, $0 < i < N$ and $\mathcal{F}(F_{i-1}) \wedge m'$ is unsatisfiable, then add $\langle m, i+1\rangle$ to Q.

Induction For $0 \leq i < N$ and a clause $(\varphi \vee \psi) \in F_i$, if $\varphi \notin F_{i+1}$, $\mathcal{F}(\phi \wedge F_i) \rightarrow \phi'$, then add φ to F_j, for all $j \leq i+1$.

until ∞;

Computing Reachable States

Successor If there is $\langle m, i+1\rangle \in Q$ and a model M $M \models \psi$, where $\psi = \mathcal{F}(\vee \textsc{Reach}) \wedge m'$. Then, add s to $\textsc{Reach}$, where $s' \in \text{MBP}(\{X, X^o\}, \psi)$.

Computing new reachable states by under-approximating forward image using MBP

- since MBP is finite, guarantee to exhaust all reachable states

Second use of MBP

- orthogonal to the use of MBP in Decide
- REACH can contain auxiliary variables, but might get too large

For Boolean CHC, the number of reachable states is bounded

- complexity is polynomial in the number of states
- same as reachability in Push Down Systems

Must and May refinement

DecideMust If there is $\langle m, i+1 \rangle \in Q$, and a model M $M \models \psi$, where $\psi = \mathcal{F}(F_i, \vee\text{REACH}) \wedge m'$. Then, add s to Q, where $s \in \text{MBP}(\{X^o, X'\}, \psi)$.

DecideMay If there is $\langle m, i+1 \rangle \in Q$ and a model M $M \models \psi$, where $\psi = \mathcal{F}(F_i) \wedge m'$. Then, add s to Q, where $s^o \in \text{MBP}(\{X, X'\}, \psi)$.

DecideMust

- use computed summary to skip over a call site

DecideMay

- use over-approximation of a calling context to guess an approximation of the call-site
- the call-site either refutes the approximation (**Conflict**) or refines it with a witness (**Successor**)

Conclusion

A program verifier is a compiler

- reusing an existing compiler is good idea, but comes with many caveats

Verification is Logic

- reduce verification to decidability of logic formulas
- CHC is a great target fragment for many verification tasks
- Greatly simplifies reasoning by discharging program semantics

An exciting direction with many extensions and open problems

- termination and model counting
- abstraction refinement and predicate abstraction
- abstract interpretation as model finding
- beyond arithmetic: arrays, memory, quantified models, separation logic, …
- program transformation for verification
- proof search strategies
- …

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